

REVIEW OF FRICTIONAL PRESSURE GRADIENT REDUCTION FOR HORIZONTAL LAMINAR PIPE FLOW OF NON- NEWTONIAN SLURRIES WITH GAS INJECTION

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The frictional pressure gradient for the horizontal, laminar pipe flow of non-Newtonian slurries can be very large. Injecting gas into the pipe immediately downstream from a pump can reduce the friction by 80% or more and occurs for the stratified, elongated bubble and slug flow patterns only. No reduction in pressure loss occurs when the flow is in the turbulent regime for either Newtonian or non-Newtonian fluids, nor is there any for a Newtonian fluid flowing in the laminar regime. As well as a reduction in frictional pressure loss, there is sometimes also an overall power saving (i.e., power to compress the gas is less than the reduction in pump power arising from gas injection), depending on the relative compressor and pump efficiencies. There is an optimal gas flowrate which achieves the greatest reduction in frictional pressure loss and this depends on several factors including the slurry flow curve, slurry flowrate, internal pipe diameter and therefore pipe Reynolds number for slurry flow alone. The greatest reduction occurs when the slurry flow alone occurs at low Reynolds numbers in the laminar flow regime. Owing to the high gas injection pressures required for long distance slurry pipelines, and the consequent large increase in air volume flowrate, this technique may be limited to short pipelines, unless a very small gas volume flowrate is injected downstream of the pump. This paper covers experimental and modelling work which identifies the conditions when this effect is most beneficial and provides industrial case studies.

KEY WORDS: pressure drop reduction, gas, non-Newtonian, horizontal pipe flow,

NOTATION

D	inside pipe diameter (m)
J	parameter defined by Equation (6) (-)
K	consistency coefficient in power law model (Pa s^n)
K'	local value of K at given shear rate (Pa s^n)
n	flow behaviour index in power law model (-)
n'	local value of flow behaviour index at a given shear rate (-)
N _{SL}	pump power for slurry flow alone (W)
P _{atm}	atmospheric pressure (Pa)
Re _{MR}	Metzner-Reed pipe Reynolds number, defined by Equation (1) (-)
Re' _{TP}	two-phase mixture power law pipe Reynolds number, based on V _m (-)
V _g	superficial air/gas velocity (m/s)
V _s	superficial velocity of viscous fluid (m/s)
V _{sc}	critical superficial fluid velocity at laminar flow breakdown (m/s)
ΔN	power saving with air injection (W)

ΔP_{TP}	frictional pressure loss for gas/fluid flow (Pa)
η_c	air compressor efficiency (-)
η_p	pump efficiency (-)
λ	input volume fraction of viscous fluid, defined by Equation (3) (-)
ρ_s	density of viscous fluid (kg/m^3)
ϕ_{s1}^2	drag ratio (-)
$(\phi_{s1}^2)_m$	minimum value of drag ratio (maximum reduction in pressure loss) (-)
ψ	coefficient of power saving, defined by Equation (7) (-)

1. INTRODUCTION

The frictional pressure loss for the flow of viscous fluids, including slurries and polymer solutions, in pipelines can be very high, but can be reduced by injecting a small air (or other gas) flowrate into the pipe immediately downstream from a pump. In some cases there is also an overall power saving, i.e., the power to compress the gas is less than the reduction in pump power arising from air injection, provided the fluid is shear-thinning and in laminar flow prior to air injection, and depending on the relative efficiencies of the pump and compressor used to inject the air. This paper includes the main benefits of air injection, outlines some simple models which predict the effect and the maximum reduction in frictional pressure loss, and includes some examples of where this pressure loss reduction has been applied in industry.

The reduction in frictional pressure loss occurs because the effect of the air presence reducing the proportion of the inner pipe wall that the fluid is in contact is greater than the effect of an increase in the mean viscous fluid pipe velocity for a fixed fluid volume flowrate. The more viscous and non-Newtonian (shear-thinning) is the pumped fluid, the greater the drag reduction effect for a given pipeline diameter and air flowrate. An air/gas injection system can be either included at the design stage to reduce pump discharge pressure and therefore pump power consumption, or retrofitted to an existing pump/pipeline system if this was initially designed for less viscous fluids and there is a need to limit the existing maximum pump discharge pressure.

The many advantages of using air injection are:

- (1) reduced pump discharge pressure for a given fluid flowrate through a given discharge pipe length, and so reduced pipe wall thickness owing to pressure reduction;
- (2) increased pipeline capacity carrying a given viscous fluid for the same pump;
- (3) pipe extension for the same pump discharge pressure and fluid volume flowrate;
- (4) use of an existing pump and pipeline combination for a more viscous, shear-thinning fluid, while maintaining the same discharge pressure;
- (5) reduced pump differential pressure, and therefore reduced slippage in some pump types, with a corresponding reduction in pump wear.

2. LITERATURE REVIEW

Pressure drop by air injection was first reported in passing by Ward and Dallavalle (1954) who used pseudohomogeneous clay suspensions. Later Oliver and Young-Hoon (1968), Mahalingham and Valle (1972), Srivatsa and Narasimhamurty (1973) and Chhabra

et al (1984) all observed pressure drop reductions for pseudoplastic polymer solutions. Dziubinski has considered pressure loss reduction in the presence of a thixotropic fluid (1988a, b). Cheng et al (1971) and Carleton et al (1973, 1975) transported asbestos slurries, bentonite pastes and concrete from a pressurised hopper by injecting air at the upstream end of the horizontal discharge line to break the material into alternate plugs of slurry and air thereby reducing the average pressure gradient along the line. Heywood et al (1976, 1978, 2004) and Farooqi et al (1980, 1982) found, when using flocculated kaolin suspensions and anthracite slurries, that for a given mean flow velocity of the viscous fluid, progressive increases in the air volume flowrate leads to a progressive reduction in pressure loss until a maximum effect is reached. Figure 1 shows the drag ratio, ϕ_{sl}^2 , defined as the ratio of the pressure gradient for the air/fluid flow divided by the pressure gradient for fluid flow alone at the same fluid volume flowrate.

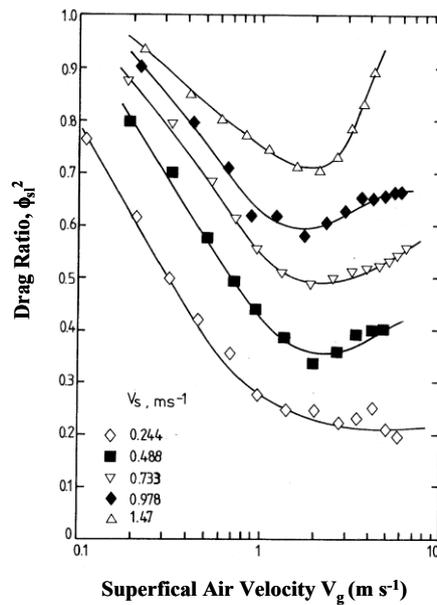


Figure 1. Typical reductions in frictional pressure loss as function of superficial fluid velocity, V_s , and superficial air velocity, V_g . (Farooqi et al, 1980)

Beyond the minimum value of ϕ_{sl}^2 corresponding to a maximum reduction in pressure loss, further increases in air flowrate lead to a progressive rise in pressure loss. At sufficiently high air flowrates the pressure loss then exceeds that for slurry flow alone.

Most research has focused on small diameter horizontal pipes. However, although the gas and liquid/slurry phases distribute in different ways, it has been demonstrated that the pressure loss reduction also occurs in large diameter pipes such as 158 mm (Heywood, 1976; Heywood and Richardson, 1978) and 207 mm (Chhabra et al, 1983).

2.1 PREDICTION OF EXTENT OF DRAG REDUCTION

A useful frictional pressure loss reduction occurs when the power law, Metzner-Reed pipe Reynolds number, Re_{MR} , for slurry flow alone is less than about 500, rather than the critical value of Reynolds number of approximately 2000. Re_{MR} is defined as

$$Re_{MR} = \frac{D^n V_s^{2-n} \rho_s}{K 8^{n-1}} \left[\frac{4n}{1+3n} \right]^n \quad (1)$$

Since the early 1970's, attempts have been made to predict the extent of the drag reduction from knowledge of the rheological behaviour of the viscous fluid and the fluid and gas flowrates. The simple plug flow model assumes that the gas and fluid move along the pipe in the form of alternate cylindrical plugs. The drag ratio can then be predicted (Carleton et al, 1973; Heywood and Richardson, 1978) using

$$\phi_{sl}^2 = \lambda^{1-n} \quad (2)$$

in which the fluid input volume fraction, the fluid holdup that would occur if the two phases moved at the same velocity in the pipe, is given by

$$\lambda = \frac{V_s}{V_s + V_g} \quad (3)$$

This equation is useful for an initial scoping design and feasibility study and is more accurate at low V_s and V_g values and high K -values in the power law model (Heywood and Richardson, 1978). In particular, the mixture Reynolds number must not exceed 500 (or, more usefully, the Fanning friction factor must be greater than $16/500 = 0.032$). The mixture Reynolds number is based on the single phase power law pipe Reynolds number:

$$Re'_{TP} = \frac{D^n (V_s + V_g)^{2-n} \rho_s}{K 8^{n-1}} \left[\frac{4n}{1+3n} \right]^n \quad (4)$$

In addition, V_g , must not exceed 1 m/s. Such conditions correspond to the plug (elongated bubble) flow pattern. The model breaks down as the maximum drag reduction is approached, and a more detailed slug flow model is required (Heywood, 1976; Farooqi et al, 1980).

Dziubinski and Chhabra (1989) used a modified Lockhart and Martinelli (1949) ϕ_{sl}^2 and a χ parameter, the square root of the ratio of the liquid/slurry phase pressure gradient to that of the gas phase pressure gradient for the flow of both phases flowing alone at their superficial pipe velocities. Both parameters were modified to take into account the superficial fluid velocity and that velocity at the point of laminar flow breakdown, together with the flow behavior index. Using a large experimental data bank, they found that the original L-M correlation could be used to estimate the two-phase pressure loss for both the

laminar and turbulent flow of the fluid phase with an error of $\pm 40\%$. Dziubinski (1995) has also proposed a semi-theoretical general method for experimental data correlation. Kaminsky (1998) suggested that his theoretical approach was expected to be more accurate for large diameter pipelines than correlations based on small-diameter pipe data. Using 696 experimental data, Xu et al (2014) compared the models of Carleton et al (1973), Heywood and Richardson (1978), Dziubinski and Chhabra (1989) and Dziubinski (1995) with a new model, and found that their model exhibited an average error of 14%. Picchi et al (2014, 2015) developed a pre-integrated two fluid model (PTF) for stratified flow and a mechanistic slug flow model. The PTF model (2014) overpredicted the drag reduction data of Xu et al (2007) for CMC shear-thinning solutions but the 2015 slug flow model gave better agreement.

2.2 MAXIMUM DRAG REDUCTION PREDICTION

The maximum pressure loss reduction (minimum drag ratio) can be correlated with Re_{MR} and parameter, n , for the power law model (Figure 2).

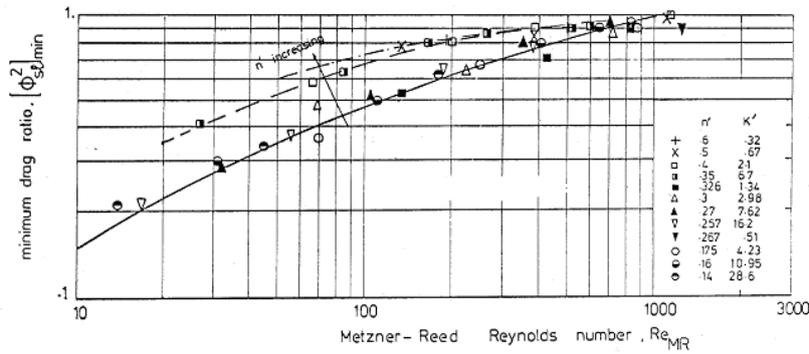


Figure 2. Maximum reductions in frictional pressure loss attainable for a power law slurry (Farooqi and Richardson, 1982)

For a given Re_{MR} , the maximum pressure loss reduction increases as n decreases. The minimum drag ratio was correlated (Farooqi and Richardson, 1982) with a dimensionless parameter J (Figure 3):

$$\begin{aligned}
 (\phi_{sl}^2)_{min} &= J^{0.205} & 0.60 < J < 1.0 \\
 (\phi_{sl}^2)_{min} &= 1 - 0.0315J^{-2.25} & 0.35 < J < 0.6 \\
 (\phi_{sl}^2)_{min} &= 1.9J & 0.05 < J < 0.35
 \end{aligned} \tag{5}$$

where

$$J = \left[\frac{V_s}{V_{sc}} \right]^{1-n} \tag{6}$$

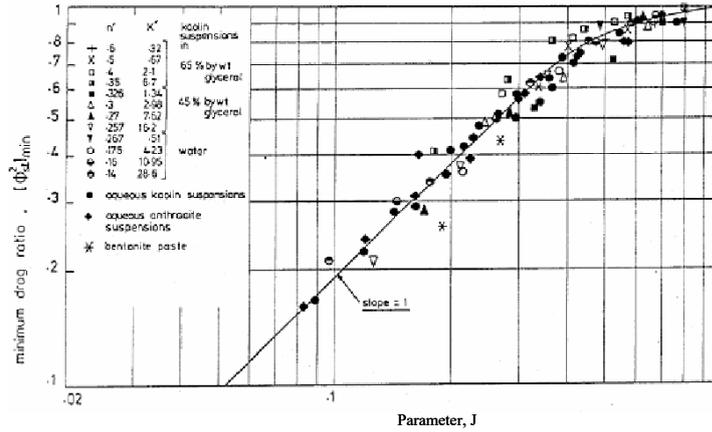


Figure 3. Relationship between minimum drag ratio and dimensionless factor J. (Farooqi and Richardson, 1982)

2.3 CONDITIONS FOR MAXIMUM POWER SAVINGS

Optimisation of air incorporation to maximise power savings has been studied (Dziubinski and Richardson, 1985; Dziubinski, 2002)). The superficial gas velocity to obtain maximum power savings was calculated and compared with experimental data. To achieve an overall power saving, the reduction in pump power from air injection must be greater than the power required to compress the air to the line pressure for pipe injection. To make the comparison, a coefficient of power saving, ψ , which normalises the power saving (ΔN) with respect to the pump power required for slurry flow alone N_{SL} is used:

$$\psi = \frac{\Delta N}{N_{SL}} \quad (7)$$

They showed that ψ could be expressed as

$$\psi = 1 - \phi_{sl}^2 - \frac{\eta_p}{\eta_c} \frac{V_g}{V_s} \left[\frac{P_{atm}}{\Delta P_s} + \frac{\phi_{sl}^2}{2} \right] \ln(1 + \Delta P_{TP}/P_{atm}) \quad (8)$$

Values of ψ were determined for a range of conditions and maximum values identified by differentiating Equation 8 with respect to V_g . Maximum values were found to occur for the plug flow pattern for all experimental data used, i.e., when $V_g < 1$ m/s and $Re_{MR} < 500$. For this flow pattern, we have already noted that the drag ratio may be expressed using Equation 2. When Equations 2 and 8 are combined, differentiated to give $d\psi/dV_g$, and this differential set to zero, this leads to the maximum power saving condition. First, it is necessary to select the desired values of V_s , n , η_p/η_c , and the ratio of the efficiencies of the

pump and air compressor, and $\Delta P_s/P_{atm}$ which is determined by the properties of the viscous fluid, its volume flowrate and the pipe dimensions.

V_g is determined by iteration. Maximum values of ψ can then be obtained from Eqn (8) and is applicable for $0.24 < V_s < 0.98$ m/s. This analysis shows that

(a) the power saving becomes progressively greater as η_p/η_c decreases; unless the efficiency of the air compressor exceeds that of the pump there will be no power saving;

(b) for typical values of pump and compressor efficiencies ($\eta_p = 0.6$; $\eta_c = 0.85$; $\eta_p/\eta_c = 0.7$), an overall power saving in gas/viscous fluid flows is possible when the flow behaviour index is below 0.3.

For significant power savings, the flow must be laminar, the flowrate must be low, and the compressor must have an appreciably greater efficiency than the pump.

3. PRACTICAL CONSIDERATIONS

There is now enough information from many experimental and modelling studies to facilitate a reliable system design. If the viscous material flow properties are characterised through viscometry, and the volume flowrate and internal pipe diameter are specified, the reduction in the pressure loss achieved for a given gas injection rate can be predicted with reasonable accuracy. Also, the maximum reduction in pressure loss and the required gas flowrate can be predicted. Because the pump discharge pressure is often quite high for some viscous fluids, injecting gas at that pressure before the pump discharge pressure is reduced could create a safety hazard. Gas injection should begin before starting the pump if the discharge pipeline is empty, or, if the pipe is already full with stationary fluid, to start the pump and inject gas concurrently and build up to the steady state pump discharge pressure. Owing to the pressure reduction in the pipe from the pump to the pipe discharge, injected gas will expand as it flows down the pipeline, accelerating the fluid along the pipe. If the material is in laminar flow prior to gas injection, two situations arise. In the first case, the pressure gradient falls along the pipe as the gas expands, and the minimum pressure gradient is achieved just before pipe discharge if the material is still in laminar flow at that point. In the second situation the maximum reduction in frictional pressure gradient occurs at some point along the pipeline, and the pressure gradient then starts to rise again until the pipe discharges.

4. INDUSTRIAL APPLICATIONS

Despite the many benefits of using gas injection into a pipe carrying non-Newtonian slurries and other viscous materials, there are few published applications. Blackwood (2018) who worked for Monsanto for many years stated that “surprisingly this technique hasn’t gained wide application”. This may be because of the lack of awareness of these benefits. De-aeration of viscous materials can also be an issue on pipeline discharge, especially for viscoplastic materials, and it might not be possible to allow fluid to become contaminated with a gas such as air, especially if the material contains easily oxidised components. However, numerous industrial examples of shear-thinning slurries pumped through pipework occur, such as red mud from bauxite processing / alumina production, PFA slurry from coal-fired power stations, titanium dioxide in paint manufacture, chalk,

clay, sewage sludges, etc. and many operations may benefit from gas injection into pipe flows.

There are relatively few published examples of applying gas injection. Cheng et al (1971) designed a system for a brake pad manufacturer for conveying asbestos slurry from a hopper through pipe work using air injection over 90 m. The system operated for at least ten years. Carleton et al (1973) devised an air injection system for bentonite pastes and modelled the flow using the simple plug flow model. Dziubinski (1988c) and Dziubinski and Fidos (1992) reported that a pipeline carrying waste molasses when processing sugar could be extended by some 50% using air injection when using the same pump and volume flowrate. Kembrowski et al (1988) studied the direct extrusion of food paste without using several plugs. Experimental data were collected using fruit jam, tomato concentrate, cacao glazing, and mayonnaise and the thickness of the remaining paste layer in the pipe was correlated with the paste Reynolds and Froude numbers. Ruiz-Viera et al (2006) studied the drag reduction effect for very viscous, non-Newtonian lubricating greases. Bjerkholt et al (2005a, 2005b) injected air into cattle and pig slurry in both a horizontal pipe and a loop reactor. No drag reduction resulted, but a trend was observed suggesting drag reduction may occur for slurries containing more than 5.5% total solids.

Putzmeister offers a “Mixokret” pumping system for transferring concrete, screed and other materials on building sites (Figure 4). It consists of a single-shaft mixer which also acts as a conveyor. The air above the material is compressed using an external compressor, pushing the mix out of the vessel. Separately compressed air is added from a second line directly to the delivery pipe, giving “air cushions” which transport the mix in an intermittent flow. A pot or curved discharge device separates the conveying air at the end allowing uniform material discharge.



Figure 4. “Mixokret” mixer/pump for building materials (Putzmeister GmbH, Aichtal, Germany)

Gabryjonczyk, (2013) suggests air injection for viscous sewage sludges. Mottyll (2018), and Mottyll(2018) and Eaton (2019) describe using air injection by the Seepex SAI (Smart Air Injection) with a boundary layer fluid (Heywood, 2003). Figure 5 shows SAI with two injection points for polymer solution and compressed air. A solid ‘plug’ of dewatered sludge is formed by an open hopper progressive cavity pump with a feed auger, and then transported using pulsed compressed air via pneumatic dense phase conveying.

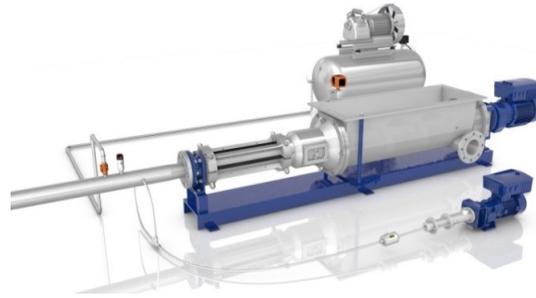


Figure 5. The Seepex SAI pumping system (Courtesy of Seepex GmbH, Germany)

The SAI system has been installed at European plants for up to one km long pipelines (Eaton, 2019). Piston pumps were replaced by SAI at the Thames Water Reading, UK, sewage treatment works, reducing energy consumption by over 50% (McGarian, 2018). 25% TS sludge is transported in 150mmID pipe over 53m at 3.5 m³/h.

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