ANOMALOUS PRESSURE DROP IN SETTLING SLURRY FLOW THROUGH PIPE OF MILD NEGATIVE SLOPE

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In an inclined pipe carrying settling slurry, the manometric pressure drop tends to increase with the increasing angle of inclination primarily due to the increasing static contribution affected by the slurry density. Our experiments show that, surprisingly enough, this trend is not continuous if a broad range of inclination angles is considered, including negative angles (i.e. descending flows). Instead, a local peak occurs in the manometric pressure drop at the angle of about – 15 degrees. The observed effect is associated with changes in solids distribution caused by a variation in the angle of inclination. In the paper, the effect is explained using measured solids distributions and using an analysis of governing mechanisms. It is demonstrated that this effect can be predicted by a layered model for inclined settling slurry flows. Model predictions of negatively sloped flows are discussed in further details, particularly predictions of descending flows near the deposition limit.

KEY WORDS: inclined flow, solids distribution, flow friction

NOTATION

c	Local volumetric concentration (-)
C_{vd}	Delivered volumetric concentration (cross-section averaged) (-)
C_{vi}	Spatial volumetric concentration (cross-section averaged) (-)
D	Inner pipe diameter (m)
g	Gravitational acceleration (m/s^2)
i	Hydraulic gradient (-)
L	Length of pipe section (m)
S	Relative density of fluid (-)
V	Flow velocity (cross-section averaged) (m/s)
y	Vertical position above bottom of pipe (m)
Δp	Pressure drop (Pa)
ρ	Density (kg/m^3)
ω	Angle of pipe inclination (rad)
Indexes:	f = carrier fluid, m = mixture, s = solids, w = standard water, ω = inclined fric = frictional, man = manometric, total = total

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1. DIFFERENT PRESSURE DROPS IN INCLINED FLOW

In pressurised flow through a horizontal pipe, the generated pressure drop is exclusively due to friction (Δp^{fric} and expressed as dimensionless hydraulic gradient $i_m^{fric} = \Delta p^{fric} / (L\rho_w g)$). In an inclined pipe, however, the total pressure drop is composed of the frictional part and the static part and it is expressed using the Bernoulli equation as

$$i_{m,\omega}^{total} = \frac{\Delta p_\omega^{total}}{L\rho_w g} = i_{m,\omega}^{fric} + S_{mi}\sin\omega \quad (1)$$

where in the static part the relative density of mixture, $S_{mi} = \rho_{mi}/\rho_w$ and it is given by $S_{mi} = S_f + (S_s - S_f)C_{vi}$. Usually, the pressure drop is measured using a differential pressure transmitter over a section of the pipe and the resulting measured manometric hydraulic gradient is

$$i_{m,\omega}^{man} = \frac{\Delta p_\omega^{man}}{L\rho_w g} = i_{m,\omega}^{total} - S_f\sin\omega = i_{m,\omega}^{fric} + (S_{mi} - S_f)\sin\omega \quad (2)$$

1.1 STATIC PART OF TOTAL PRESSURE DROP

The static pressure drop due to inclination depends on the in situ concentration Cvi but usually predictive pressure drop formulae require delivered concentration Cvd as an input. Determination of the difference between Cvi and Cvd due to slip between carrier liquid and solid particles is an important part of the evaluation of different types of pressure drop in inclined flows. In settling slurry flow, this difference is usually not negligible in the range of operational velocities provided that the flow is not inclined to very steep angles close to the vertical. Note that the static part of the total pressure drop can be significantly bigger than the frictional part in inclined flows even at mild slopes, particularly for flows of settling slurry at volumetric concentrations typical for practical applications.

1.2 PREDICTION OF INCLINATION EFFECT ON PRESSURE DROP

A Worster-Denny formula (Worster and Denny, 1959; Wilson et al., 2006) is a simple and widely-used tool for prediction of the effect of pipe inclination on the pressure drop of settling slurry flows. It is based on the following considerations. In horizontal flows, the hydraulic gradient contribution generated by solids is simply the frictional solids effect (i_m^{fric} - i_f). In inclined flows, the frictional solids effect is associated with the cross-pipe component of the submerged weight, so that (i_m^{fric} - i_f)cosω contributes to the total gradient by solids friction. A further contribution is the solids effect on the static gradient in the vertical projection of the inclined pipe, which is assumed to be $(S_s - S_f)C_{vd}\sin\omega$. From this description of the pressure drop contributions, it follows that

$$\frac{\Delta p_\omega^{man}}{\rho_w g L} = i_f + (i_m^{fric} - i_f)\cos\omega + (S_s - S_f)\,C_{vd}\sin\omega \quad (3)$$

Note that Equation 3 predicts the same frictional pressure drop in the ascending pipe and in the descending pipe of the same inclination angle (cos(-ω) = cosω). Furthermore, it neglects the effects of slip and of a possible variation in the flow pattern/regime with flow inclination.

Alternatively, the different pressure drops can be predicted by a layered model for inclined settling slurry flows (e.g. Matousek et al., 2018).

2. EXPERIMENTALLY OBSERVED PRESSURE DROP IN INCLINED FLOW

Inclined flow experiments were carried out for flow of sand-water mixtures in the 100-mm loop with an inclinable inverted U-tube at the Institute of Hydrodynamics in Prague. The loop, its measuring equipment, measuring techniques and experimental procedures for inclined flow tests are described in Matousek et al. (2018).

We report results of the experiment with the 0.55 mm sand (narrow graded sand with the mean grain size of 0.55 mm and grain density of 2597 kg/m^3) at mean delivered concentration (C_{vd}) of about 0.24 and inclinations of ± 45 degrees from horizontal. Measurements included mean flow velocity V_m, manometric pressure drops in ascending and descending limbs of the U-tube, the delivered concentration C_{vd} and chord-averaged vertical concentration distributions in pressure-drop measuring sections of both limbs of the U-tube. In a test run, V_m and C_{vd} are the same in both measuring sections irrespective of the inclination of the U-tube. The solids distribution (and hence C_{vi} obtained by an integration of the solids distribution) and the manometric pressure drop should be the same in the U-tube set to the horizontal position. If the U-tube is inclined to any angle different from zero, then values of these parameters are different in the two sections.

2.1 MEASURED MANOMETRIC PRESSURE DROP

Figure 1 shows measured manometric pressure drop (hydraulic gradient) at approximately constant V_m (2.5 m/s) in the U-tube inclined to different angles from 0 up ± 45 degrees. The experimental results confirm a general trend of an increase in the manometric hydraulic gradient with the increasing angle of inclination due to the increasing static part of the gradient. However, the results also show a local deviation from this trend for negatively sloped flows at ω between say -5 to – 25 degrees.

Anomalous pressure drops occur with a local peak at -15 degrees where the measured manometric hydraulic gradient is considerably higher than the general trend suggests and also considerably higher than the predictions by Worster-Denny formula (Equation 3) suggest. Figure 1 shows the predictions using Equation 3 for measured values of C_{vd} at different ω and also the predictions for which C_{vd} was replaced by the measured C_{vi} at each ω in Equation 3. The use of C_{vi} instead of C_{vd} slightly improves an ability of Equation 3 to predict the effect of flow inclination at $\omega > 0$ degree but is unable to follow the trend measured in negatively sloped flows ($\omega < 0$ degree).

The frictional hydraulic gradients are plotted as well in Figure 1. The experimental frictional gradient is obtained by subtracting the C_{vi}-based static part of the manometric gradient and it shows that the anomalous trend in the development of the hydraulic gradient is predominantly due to friction, not due to the static gradient. The frictional gradient obtained from Equation 3 is unable to capture the trend as it predicts the same frictional gradient in the ascending and descending pipe of the same absolute value of ω as discussed above.

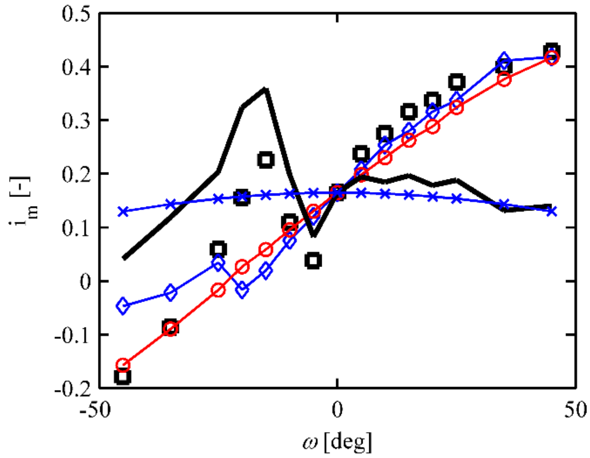

Figure 1. Different hydraulic gradients (dimensionless pressure drops) in sand-water flow at $V_m \approx$ 2.5 m/s and $C_{vd} \approx 0.24$ for various flow inclinations. Legend: black square marker = measured manometric gradient, black full line = frictional gradient from measurement, red o-line = manometric gradient by Worster-Denny (Equation 3) based on C_{vd}, blue diamond-line = manometric gradient by Worster-Denny (Equation 3) based on C_{vi}, blue x-line = frictional gradient by Worster-Denny

2.2 STATIC PART FROM EXPERIMENT

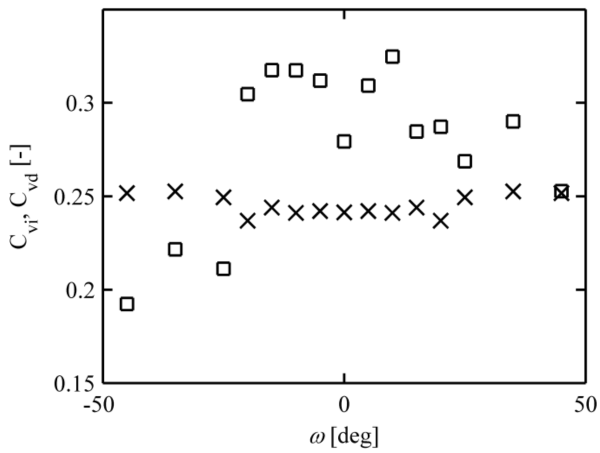

Figure 2. Experimentally determined mean volumetric concentrations in sand-water flow at $V_m \approx$ 2.5 m/s and $C_{vd} \approx 0.24$ for various flow inclinations. Legend: square marker = spatial concentration C_{vi}, x-marker = delivered concentration C_{vd}.

Figure 2 compares measured C_{vd} and C_{vi} at various flow inclination angles and enables quantification of the difference. The difference (and hence the measure of mean slip) is non-negligible in the horizontal flow and in flows of mild slopes (both positive and negative). In ascending flow, the difference (C_{vi}-C_{vd}) tends to zero at $\omega > 45$ degrees indicating that mean slip is negligible in steep ascending flow. In descending flow, (C_{vi}-C_{vd}) reaches negative values at $\omega > 20$ degrees suggesting that $C_{vi} > C_{vd}$ and solid grains move faster than carrying liquid. This point will be discussed below in connection with the solids distribution in descending flow.

2.3 FLOW STRATIFICATION AND SOLIDS DISTRIBUTION

Measured solids distributions in the tested sand slurry flow demonstrated that the horizontal flow was actually very weakly stratified with virtually no *en-bloc* sliding bed in both measuring sections of the U-tube set to a horizontal position (Figure 3, left plot). The solids distribution was almost linear across the entire cross-section of the pipe in both sections. However, if the U-tube was tilted to ± 15 degrees, the conditions in the two sections became different. The solids distribution remained very similar to the horizontal flow in the ascending flow while it changed significantly in the descending flow (Figure 3, right plot). In the descending pipe, the degree of stratification increased and virtually fully stratified flow developed with a thick sliding bed. Very similar shapes of concentration profiles were measured also at inclination angles of 5, 10, 20, 25, and 35 degrees (Matousek et al. 2019). This indicated that the negative slip at $-45 < \omega < -20$ degrees (Figure 2) was associated with fast grains transported as the sliding bed. Apparently, the bed slid faster than the carrying liquid flowed above the bed. Hence, also contact-load grains in flow at -15 degrees (Figure 3) moved faster than contact-load grains (grains at the bottom of the pipe) in the horizontal flow.

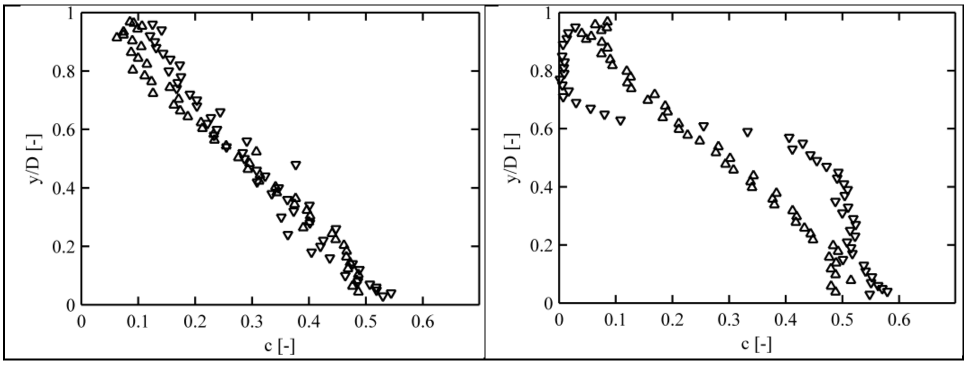

Figure 3. Measured solids distributions in sand-water flow at $V_m \approx 2.5$ m/s and $C_{vd} \approx 0.24$ at inclination angle 0 degree (left plot) and ± 15 degrees (right plot). Legend: ^ = ascending pipe section of inclinable U-tube, v = descending pipe section of U-tube

3. DISCUSSION OF DETECTED ANOMALOUS PRESSURE DROP IN DESCENDING FLOW

As shown above, the flow inclination affects the pressure drop (Figure 1) through the non-negligible slip and its variation with the inclination angle (Figure 2) but apparently also through a variable frictional pressure drop. In the descending flows of mild slopes (angles between say -5 and -25 degrees), this frictional effect dominates and becomes significantly more important than the static pressure effect.

3.1 MECHANISM OF ANOMALOUS PRESSURE DROP

At the inclination angles bigger than say -35 degrees, descending flow exhibits significantly stronger stratification than ascending- or horizontal flow (Figure 3). This trend was observed also in coarser sand fractions tested earlier (Matousek, 1996; Matousek et al., 2018). In ascending flows, weak stratification was attributed to the shearing off of the top of the slow sliding bed in the pipe. In the descending flows, particularly between -10 and -35 degrees the sliding bed was not sheared off because the difference in velocities of the sliding bed and the flow above the bed was small. The bed started to disintegrate at steep inclination angles. For the medium sand discussed in this paper, first signs of the disintegration can be observed on the shape of the profile at -45 degrees (Matousek et al., 2019).

The experimental results reveal that the anomalous friction is closely related to the variation in the flow stratification (i.e. variation in shapes of concentration profiles) with pipe inclination angle. The relation between flow friction and degree of solids stratification can be simply explained by principles of force balance (a balance between driving and resisting forces) applied to two layers of the fully- and partially-stratified flow as exploited in layered models.

In the descending pipe, the bed slides faster than in the horizontal pipe due to the bed submerged-weight component acting as an additional driving force in the flow direction. The top of the faster sliding bed is less eroded and hence the flow more stratified and the sliding bed thicker. At the same time, the more stratified flow exerts more resistance due to the bigger normal component of bed submerge weight and the dominating effect of mechanical sliding friction on the frictional pressure drop. The fast sliding bed of strongly stratified descending flow produces significant solids friction and thus demands a sufficiently strong driving force to push the bed. This force is produced predominantly by the pressure drop at mild inclinations and by the longitudinal component of the bed submerged weight at steeper slopes. The submerged-weight component normal to the flow direction decreases with increasing slope of descending pipe.

At mild negative slopes, the slope-related increase in the bed resisting force due to stronger flow stratification overrules the increase in the driving force from the submerged-weight component acting in the flow direction. The friction effect becomes ineffective at steeper negative slopes where the submerged-weight component in the flow direction becomes big enough to drive the sliding bed and to overcome the solids friction.

3.2 CAPABILITIES TO PREDICT ANOMALOUS PRESSURE DROP

Figure 4 compares the experimentally determined frictional gradient with predictions by Worster-Denny method (Equation 3) and by our layered model (Matousek et al. 2018). In the descending flows, in the region of anomalous pressure drop, the model is more successful than the Worster-Denny method as it is able to capture at least the trend leading towards the local peak at one of the negative slopes. At this stage, the layered model is not able to handle stratified flows in which the lower layer (the sliding bed) moves faster than the upper layer. Hence, no predictions could be carried out for $\omega < -20$ degrees. More about the layered-model predictions of this flow is in Matousek et al. (2019).

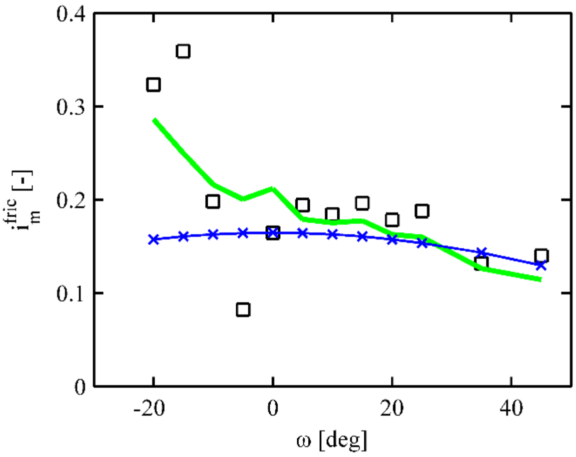

Figure 4. Frictional hydraulic gradient in sand-water flow at $V_m \approx 2.5$ m/s and $C_{vd} \approx 0.24$ for various flow inclinations. Legend: black square marker = experiment, blue x-line = prediction by Worster-Denny, green full line = prediction by layered model

4. CONCLUSIONS

Descending partially-stratified flows can exert much higher friction than expected (and conventionally predicted) in pipes inclined to angles less steep than say -25 degrees. The frictional pressure drop can be bigger than in a horizontal pipe. The reason is a development of virtually fully stratified flow sliding over the bottom of the descending pipe compared to partially stratified flow of the same slurry in the horizontal pipe. The flow becomes fully stratified in the descending pipe because the bed slides faster (due to the submerged-weight component acting as an additional driving force in the flow direction) than in the horizontal pipe. The top of the faster sliding bed is less eroded and hence the flow more stratified. At the same time, the more stratified flow exerts more resistance.

The sliding bed of strongly stratified descending flow produces significant solids friction and thus demands a sufficiently strong driving force to push the bed. This force is produced predominantly by the pressure drop at mild inclinations (to say -20 degrees) and by the longitudinal component of the bed submerged weight at steeper slopes. At the steep negative slopes, the same force is responsible for the negative slip as the bed slides faster that liquid flows above the bed.

In this paper, the described flow phenomena are demonstrated by experimental results for the flow of a sand-water mixture (narrow-graded sand of 0.55 mm grain size) of delivered concentration 0.24 and flow velocity 2.5 m/s in a 100 mm pipe inclined to ±45 degrees.

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