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# SCALE LAWS FOR SPILLAGE IN CUTTERHEADS IN DREDGING

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In dredging soil is excavated with dredging equipment. One of the main types of equipment is the cutter suction dredge (CSD). The CSD consists of a floating pontoon, with in the back a spud pole penetrating the soil. In the front there is a ladder, which can rotate around a horizontal bearing. By means of this rotation the cutter head, mounted at the end of the ladder, can be positioned in the bank. Also, at the end of the ladder two swing wires are connected (port and starboard wires) enabling the CSD to rotate around the spud pole and thus letting the cutter head make a circular movement through the bank. During this rotation, with a circumferential swing velocity vs at the centre of the cutter head, the cutter head (also rotating around its axis with a certain rpm) is excavating the soil. The theoretical soil production $Q_c$ equals the cross section of the cutter head in the bank cutting, perpendicular to the swing velocity $v_s$ times the swing velocity $v_s$. The cutter head consists of the cutter axis connected to the hub, 5 or 6 arms on one side connected to the hub and on the other side connected to the ring and a suction pipe to catch the soil cut and transport the soil to its destination. The difference between the theoretical production and the real production is the spillage. So, this is the percentage of the theoretical production not entering the suction pipe.

KEY WORDS: Dredging, Spillage, Cutterhead, Sand, Gravel, Rock.

## NOMENCLATURE

| | | |
|---|---|---|
| Bu | Den Burger dimensionless number | - |
| $C_{vs}$ | Spatial volumetric concentration | - |
| $D_r$ | Cutter ring diameter | m |
| f | Radii factor | - |
| n | Porosity | - |
| $\Delta p_E$ | Euler pressure difference | kPa |
| $P_c$ | Percentage circumference involved in cutting (as a factor) | - |
| $P_{c,1}$ | Percentage circumference involved in cutting (as a factor) segment 1 | - |
| $P_{c,2}$ | Percentage circumference involved in cutting (as a factor) segment 2 | - |
| Q | Flow | $m^3/s$ |
| $Q_a$ | Axial flow | $m^3/s$ |
| $Q_c$ | Cut production situ soil | $m^3/s$ |
| $Q_s$ | Cut production solids | $m^3/s$ |
| $Q_m$ | Mixture flow suction mouth | $m^3/s$ |
| $Q_{1,out}$ | Mixture outflow segment 1 | $m^3/s$ |
| $Q_{2,in}$ | Mixture inflow segment 2 | $m^3/s$ |
| $r_o$ | Outer radius | m |
| $r_i$ | Inner radius | m |

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| | | |
|---|---|---|
| $r_{o,1}$ | Outer radius segment 1 | m |
| $r_{i,1}$ | Inner radius segment 1 | m |
| $r_{o,2}$ | Outer radius segment 2 | m |
| $r_{i,2}$ | Inner radius segment 2 | m |
| $r_r$ | Cutter ring radius | m |
| $u_o$ | Circumferential velocity outer radius | m/s |
| $u_i$ | Circumferential velocity inner radius | m/s |
| $v_s$ | Swing speed | m/s |
| $v_t$ | Terminal settling velocity particles | m/s |
| w | Width (or height) of cutter head | m |
| $w_{1/2}$ | Width segment 1/2 | m |
| $\alpha$ | Flow factor | - |
| $\beta_o$ | Blade angle outer radius | rad |
| $\beta_i$ | Blade angle inner radius | rad |
| $\rho_m$ | Mixture density | ton/m$^3$ |
| $\omega$ | Radial frequency cutter head | rad/s |
| $\xi$ | Factor in FD (filling degree) dimensionless number | - |
| $\theta$ | Ladder angle | rad |
| $\lambda_l$ | Length scale | - |

## 1. INTRODUCTION

In dredging soil is excavated with dredging equipment. One of the main types of equipment is the cutter suction dredge (CSD). Not all the soil that is excavated with the cutterhead, will enter the suction mouth. The amount that does not enter the suction mouth is named spillage and is often used as a percentage of the theoretical production. Mol (1977A), (1977B) and Moret (1977A), (1977B) were of the first to investigate spillage. Miltenburg (1982) carried out numerous experiments with a 400 mm model cutterhead. In the next decades den Burger (2001), (2003), den Burger & Talmon (2001), (2002), den Burger et al. (2005), (1999) and Talmon et al. (2010) investigated spillage in rock cutting. This resulted in qualitative understanding, but not yet in quantitively modelling. The scale laws applied were based on the Euler and the Froude number and sometimes the Reynolds number, but not on the physics of the spillage process.

Lately Miedema (2019) developed an analytical model for spillage based on the Euler equation for centrifugal pumps. Based on this model scale laws are derived. Since the model is based on physics, the scale laws are based on the physics and not on dimensionless numbers. In fact, new dimensionless numbers are derived based on the scale laws.

Figure 1 shows the cutting process of a CSD cutting rock with production in the suction pipe and spillage in the consolidated spillage layer.

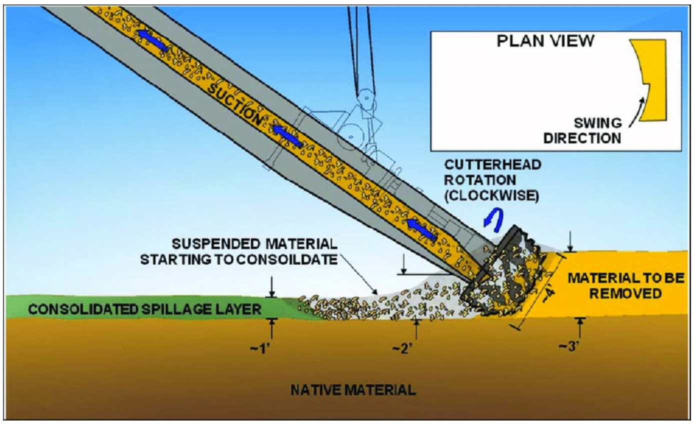


Figure 1. Spillage of a cutterhead (Fuglevand & Webb **Invalid source specified.**).

## 2. THE MODEL

The flows out of segment 1 is (see Figure 4):

$$Q_{1,out}=\alpha\cdot 2\cdot\pi\cdot\omega\cdot r_{o,1}^2\cdot\left(\frac{f}{(1+f)}\cdot w-\frac{1}{(1+f)}\cdot\frac{1}{2\cdot\pi\cdot\alpha\cdot\omega}\cdot\left(\frac{Q_m-Q_c-Q_a}{r_{o,1}^2\cdot(1-P_{c,1})}\right)\right)\cdot(1-P_{c,1}) \tag{1}$$

The flow into segment 2 is (see Figure 4):

$$Q_{2,in}=2\cdot\pi\cdot\alpha\cdot\omega\cdot\left(r_{o,1}^2-r_{o,2}^2\right)\cdot\left(\frac{1}{(1+f)}\cdot w+\frac{1}{(1+f)}\cdot\frac{1}{2\cdot\pi\cdot\alpha\cdot\omega}\cdot\left(\frac{Q_m-Q_c-Q_a}{r_{o,1}^2\cdot(1-P_{c,1})}\right)\right)\cdot(1-P_{c,2}) \tag{2}$$

To incorporate the gravity and mixing effect the following equation is found for the advanced and preliminary models:

$$Spillage=\frac{Q_{1,out}\cdot\left(C_{vs}+\left(C_{vs,max}-C_{vs}\right)\cdot\left(0.1\cdot\left(\frac{v_t\cdot sin(\theta)\cdot\pi\cdot r_r^2}{Q_m}\right)^2+\left(\frac{Bu}{10.8}\right)^3-\left(\frac{Bu}{12}\right)^4\right)\right)}{Q_s} \tag{3}$$

$$With:\quad C_{vs,max}=\frac{Q_s}{Q_{1,out}}$$

The maximum concentration is limited to a value between 0.5 and 0.6, since this gives solid sand. Of course, spillage can only occur with particles that have entered the cutter head, so the filling degree is defined as:

$$FillingDegree = \xi \cdot \left( \frac{\frac{D_r \cdot 2 \cdot \pi \cdot n}{2 \cdot 60} \cdot cos(\theta)}{v_t} \right)^2 \quad and \quad FillingDegree \leq 1 \tag{4}$$

With ξ=0.15. The final spillage can now be determined with:

$$FinalSpillage = Spillage \cdot FillingDegree + (1 - FillingDegree) \tag{5}$$

To match the experiments of den Burger, the factor $\varepsilon$ is about 2.45 for sand and 4.4 for rock. This can be described by:

$$\varepsilon = 2.35 + (4.40 - 2.35) \cdot \frac{v_t}{0.45} \cdot \lambda^{-0.4} \tag{6}$$

## 3. SCALING LAWS

1. The ladder angle must be the same in prototype and model.
2. The part of the cross section of the cutter head cutting must have the same shape in prototype and model.
3. The volumetric concentration in cutter head and suction mouth must be the same in prototype and model. This relates the swing velocity times the cross section cutting (the cut production) to the mixture flow through the suction mouth.

$$\frac{Q_c \cdot (1-n)}{Q_m} = cons\tan t \tag{7}$$

4. The ratio of the rotating mixture flow to the mixture flow through the suction mouth must be constant.

$$Bu = \frac{\omega \cdot r_r^3}{Q_m} = cons\tan t \tag{8}$$

5. The ratio of the settling flux through a cutter head cross section to the mixture flow through the suction mouth must be constant.

$$\frac{v_t \cdot r_r^2}{Q_m} = cons\tan t \tag{9}$$

The dimensionless number based on the filling degree must be constant:

$$\frac{\omega \cdot r_r}{v_t} = constant \tag{10}$$

If the conditions of Equations 8 and 9 are met, automatically Equation (10) is valid. So, basically there are 5 independent scaling rules that have to be met. Now how to use the scale laws? Where to start? Let's assume the model cutter head has exactly the same shape as the prototype cutter head and there is a length scale $\lambda_l$. The same shape also means that the cross section of the suction mouth scales with the length scale squared. The mixture velocity scales roughly with the length scale to the power 0.4 (see Miedema (2016)). This is based on the scaling of the Limit Deposit Velocity of settling slurries. So, the mixture flow scales according to:

$$\frac{Q_{m,p}}{Q_{m,m}} = \lambda_l^{2.4} \tag{11}$$

This means, with Equation 7 that the cut production has to scale in the same way, so:

$$\frac{Q_{c,p}}{Q_{c,m}} = \lambda_l^{2.4} = \frac{A_{c,p} \cdot v_{s,p}}{A_{c,m} \cdot v_{s,m}} = \lambda_l^2 \cdot \lambda_l^{0.4} \quad \Rightarrow \quad \frac{v_{s,p}}{v_{s,m}} = \lambda_l^{0.4} \tag{12}$$

This results in a swing speed that scales in the same manner as the mixture velocity, because the mixture flow scales the same way as the cut production, assuming the porosity of the sand or gravel is constant. For the dimensionless den Burger number this gives:

$$\frac{Bu_p}{Bu_m} = \frac{\dfrac{\omega_p \cdot r_{r,p}^3}{Q_{m,p}}}{\dfrac{\omega_m \cdot r_{r,m}^3}{Q_{m,m}}} = \frac{\omega_p}{\omega_m} \cdot \frac{r_{r,p}^3}{r_{r,m}^3} \cdot \frac{Q_{m,m}}{Q_{m,p}} = \frac{\omega_p}{\omega_m} \cdot \lambda_l^3 \cdot \lambda_l^{-2.4} = 1 \quad \Rightarrow \quad \frac{\omega_p}{\omega_m} = \lambda_l^{-0.6} \tag{13}$$

Using the settling flux to mixture flow ratio the following is found:

$$\frac{\dfrac{v_{t,p} \cdot r_{r,p}^2}{Q_{m,p}}}{\dfrac{v_{t,m} \cdot r_{r,m}^2}{Q_{m,m}}} = \frac{v_{t,p}}{v_{t,m}} \cdot \frac{r_{r,p}^2}{r_{r,m}^2} \cdot \frac{Q_{m,m}}{Q_{m,p}} = \frac{v_{t,p}}{v_{t,m}} \cdot \lambda_l^2 \cdot \lambda_l^{-2.4} = 1 \quad \Rightarrow \quad \frac{v_{t,p}}{v_{t,m}} = \lambda_l^{0.4} \tag{14}$$

Checking the latter with the filling degree parameter gives:

$$\frac{\dfrac{\omega_p \cdot r_{r,p}}{v_{t,p}}}{\dfrac{\omega_m \cdot r_{r,m}}{v_{t,m}}} = \frac{\omega_p}{\omega_m} \cdot \frac{r_{r,p}}{r_{r,m}} \cdot \frac{v_{t,m}}{v_{t,p}} = \lambda_l^{-0.6} \cdot \lambda_l^1 \cdot \lambda_l^{-0.4} = 1 \tag{15}$$

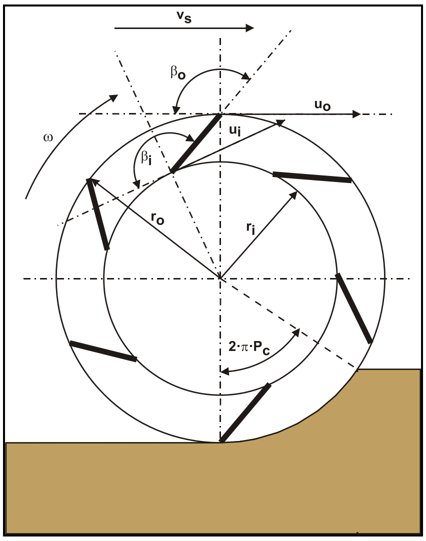


Figure 2. The definitions of the Euler equation for a cutter head.

The latter shows that the scale laws are consistent. All 3 velocities, the mixture velocity, the swing velocity and the terminal settling velocity, scale with the length scale to a power of 0.4. This also implies that in prototype larger particles are required than in model. The cutter head revolutions scale with the length scale to a power of minus 0.6, meaning the revolutions of the model are higher than the revolutions of the prototype.

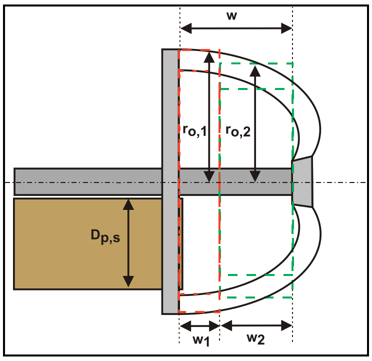


Figure 3: Cutterhead segments.

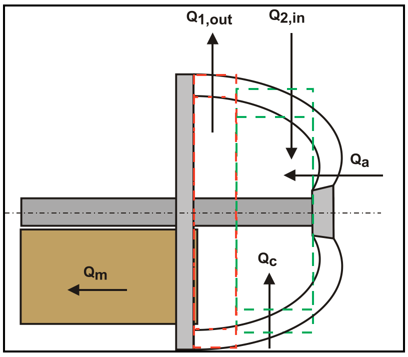


Figure 4: Cutterhead flows.

## 4. VALIDATION DEN BURGER (2003) AND MILTENBURG (1983)

Comparing this with Figure (5), the model rules as applied by den Burger (2003), a length scale of about 7.8 was used.

**Table 6.1:** Properties on prototype and model scale

| | prototype scale | model scale |
|---|---|---|
| Diameter suction pipe: $D_{sp}$ | 0.95 [m] | 0.1 [m] |
| Diameter ring cutter head: $D_c$ | 3.12 [m] | 0.4 [m] |
| Density rock: $\rho_r$ | 2200 [kg/m$^3$] | |
| Density gravel bank: $\rho_b$ | | 1700 [kg/m$^3$] |
| Density gravel grain: $\rho_g$ | | 2650 [kg/m$^3$] |
| Suction flow: $Q_s$ | 3.0 [m$^3$/s] | 0.021 [m$^3$/s] |
| (mixture velocity: $v_m$) | (4.2 m/s) | (2.6 m/s) |
| Rotational velocity: $n_c$ | 30 [RPM] | 90 [RPM] |
| Haul velocity: $v_h$ | 0.2 [m/s] | 0.1 [m/s] |
| Cut off area: $A_{cut}$ | 1.4 [m$^2$] | 0.023 [m$^2$] |
| Cutter inclination angle: $\lambda$ | 45 [°] | 45 [°] |

Figure 3. Model and prototype of den Burger (2003)

Figure 4. The crown cutter head used by Miltenburg (1982).

This should result in a mixture flow ratio of 138.4, while 143 was used. So almost the same. The revolutions of the model should be 3.43 times the revolutions of the prototype, this was a factor 3, so also close. The swing velocity in prototype should be 2.27 times the swing velocity in the model, which was a factor 2, so again close. For the terminal settling velocity no scaling was reported. However, according to the above this should be a factor 2.27, similar to the swing speed ratio. So, the conclusion is that the den Burger (2003) scale laws were close to the scale laws derived here, with the exception for the scale law for the terminal settling velocity, which was not present in den Burger (2003). Miltenburg (1982) carried out experiments in 1983 with the same cutterhead (see Figure 6) as den Burger (2003). The resulting upper and lower limit spillage/production curves match very well with the experimental data see Figure 7.

## 5. CONCLUSIONS

The analytical model and the scale laws that the paper puts out are reasonable, they matched very well with the experiments carried out by Miltenburg (1982) and den Burger (2003), and will be tested and verified further.

The experiments of both den Burger (2003) and Miltenburg (1982) were scaled based on Froude, velocity to the power 0.5. Here velocity to the power 0.4 is derived. Experiments at different scales are recommended for future research.

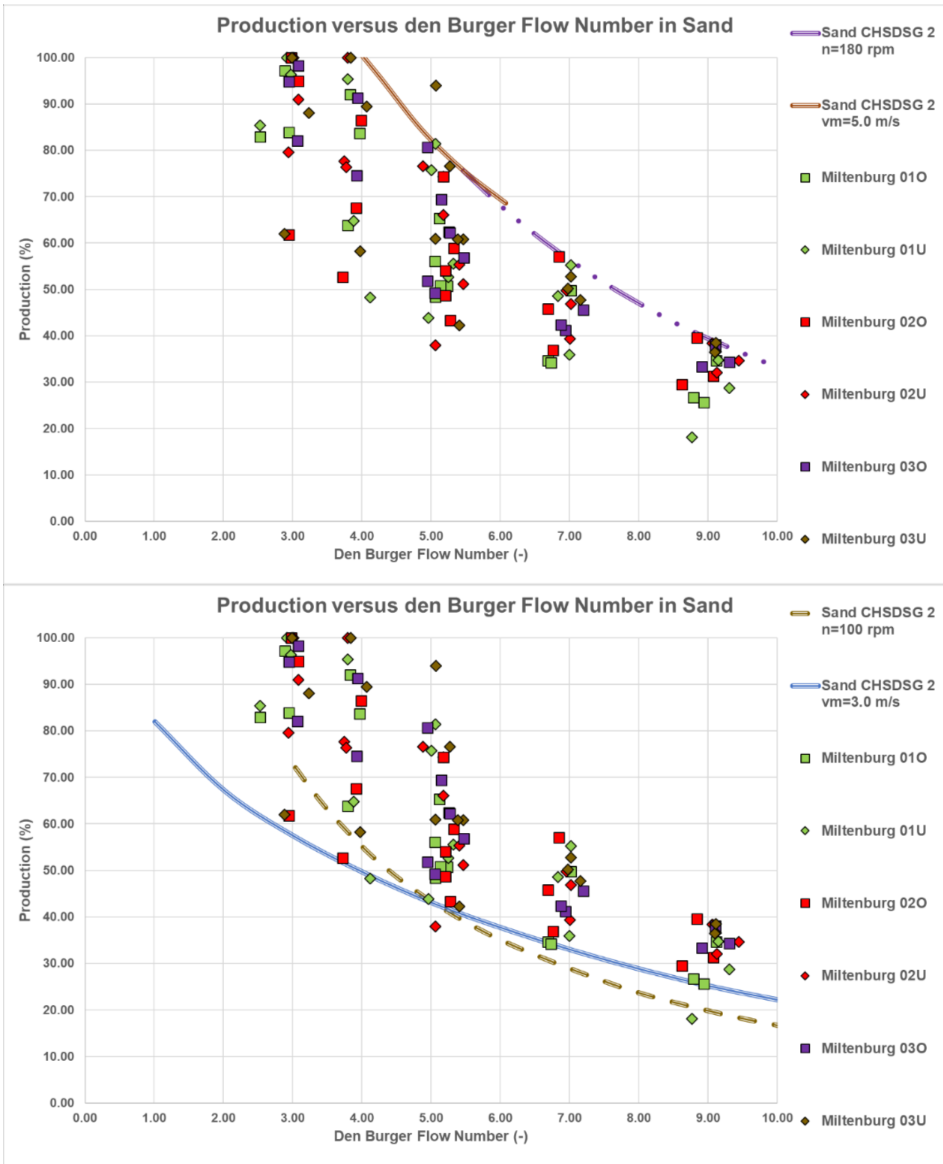


Figure 5. Experiments of Miltenburg (1982) with a rock cutter head in sand, upper and lower limit.

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