19th International Conference on
TRANSPORT AND SEDIMENTATION OF SOLID PARTICLES
24-27 September 2019, Cape Town, South Africa

DEPOSITION OF WIDE SIZE DISTRIBUTION, BINGHAM PLASTIC SLURRIES IN TURBULENT PIPE FLOW

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At the 17th T&S conference in Delft, the author (Thomas, 2015), presented a Modified Wilson & Judge (MW&J) method for predicting the deposit velocity of mono-sized particles in Newtonian fluids. The present paper extends consideration of the MW&J method to wide size distribution mineral slurries, which have Bingham plastic properties. For these slurries, turbulent flow deposition can occur due to three processes: heterogeneous deposition as predicted by the MW&J method; viscous sub-layer deposition which occurs when particles are smaller than the viscous sub-layer; and transition deposition where deposition is due to transition to laminar flow. The three types of deposition are investigated primarily by comparing predictions with the test loop data of Goosen & Paterson (2014). The Goosen & Paterson data applies to a wide size distribution gold tailings with Bingham plastic properties. The paper discusses the relevant viscosity to use in the MW&J and the viscous sub-layer predictions, and two options are compared. The first option simply uses the plastic viscosity and the second option uses the effective viscosity which varies with velocity. The -75 µm portion is assumed to represent a homogeneous vehicle slurry, and the +75 µm portion is assumed represented by the 110 µm, d_{50} size, of the +75 µm portion. At very high concentrations, and especially in the two smaller pipe diameters, laminar flow without deposition was observed in the Goosen & Paterson data. Laminar flow in test loops and in longer pipelines is discussed.

KEY WORDS: slurry pipeline, Bingham plastic, deposition, deposit velocity

1. INTRODUCTION

The author, Thomas (2014), showed how the majority of slurries pumped in the mining industry have a wide particle size distribution, are relatively fine, and possess a viscosity higher than the viscosity of water. They flow pseudo-homogeneously in turbulent flow, until deposition occurs. Pressure gradient prediction therefore depends mostly on selection of the operating velocity which must be a suitable margin above the deposit velocity (V_d), which is defined as the velocity at which a stationary bed of solids first appears as the velocity is progressively reduced. These slurries typically have a d_{50} particle size less than 100 µm. Because the viscosity is higher than that of water, the particle settling velocity of a 100 µm d_{50} size particle is equivalent to a much finer sand particle in water. For example a d_{50} = 100 µm particle could be equivalent to a 50 µm or 60 µm sand particle in water. The Wilson & Judge (W&J), 1976 deposit velocity correlation is of limited use for such

ISSN 0867-7964 ISBN 978-83-7717-323-7

fine equivalent sand particles and so is of limited use for the majority of wide size distribution slurries pumped in the mining industry.

Thomas (2015) recognised this problem and presented a Modified Wilson & Judge (MW&J) prediction method which extended the W&J (1976) correlation to finer particles and larger pipe sizes and is therefore more suitable for the typical slurries pumped in the mining industry. However the Thomas (2015) paper did not consider in any detail, the application of the MW&J prediction method to wide size distribution, Bingham plastic slurries. The current paper applies the Thomas (2015) MW&J prediction method to a typical, fine particle, wide size distribution slurry possessing Bingham plastic properties, with predictions being compared with the loop test data of Goosen and Paterson (2014).

2. MW&J PREDICTION METHOD AND ITS APPLICATION TO WIDE SIZE DISTRIBUTION SLURRIES

The MW&J prediction method presented by Thomas (2015) applied to mono-sized particles in water or in a Newtonian fluid. One method of analysing a wide particle size distribution slurry is to split the slurry into a "vehicle" component and a coarser component considered to be transported in the vehicle slurry. The vehicle slurry is assumed to act as a homogeneous fluid with density and viscosity greater than water. The MW&J equations of Thomas (2015) therefore need to be recast in terms of the density of the particles (ρ_p) and the density of the vehicle slurry (ρ_{veh}). In the current paper the MW&J deposit velocity is denoted $V_{d\Delta}$ because of the Δ term in Equation 2.

$$V_{d\Delta} = F_L \; [2gD(\rho_p - \rho_{veh})/\rho_{veh}]^{0.5} \qquad (1)$$

where D is the pipe diameter and F_L is given by:

$$F_L = 2 + 0.305 \; \log_{10}\Delta + 1.1\text{x} \; 10^{-4} \; \Delta^{-0.489} - 0.044 \; (1\text{x}10^7 \; \Delta)^{-1.06} \qquad (2)$$

where $$\Delta = 0.75 \; W^2/[gD(\rho_p - \rho_{veh})/\rho_{veh}] \qquad (3)$$

W = calculated particle terminal settling velocity of a relevant particle of size (d) in a quiescent fluid with density and viscosity equal to the vehicle slurry.

In their separate analysis, of their data, Goosen and Paterson (2014) assumed the vehicle slurry consisted of particles finer than 75 μm, and the same approach will be adopted in this paper. For the gold tailings slurry tested by Goosen and Paterson, 73% of the particles are finer than 75 μm. The d50 size of coarse (+75 μm) solids is 110 μm, and this is used as the representative size for the heterogeneous (MW&J) predictions.

For the slurries of interest in this paper it is common for the rheology (plastic viscosity and yield stress) of the total slurry to be measured, and this was the procedure followed by Goosen and Paterson. With knowledge of the rheology of the total slurry, a method of predicting the rheology of the -75 μm vehicle slurry is therefore required. In this paper the equation derived by Thomas (2010) is adapted for the purpose.

Thomas (2010) investigated the increase in plastic viscosity of clay slurries due to the addition of sand particles. In the present context, the +75 µm solids represent the sand particles, and the -75 µm vehicle slurry represents the clay slurry. The following equation therefore relates the plastic viscosity of the total slurry (η_{tot}) to the plastic viscosity of the -75 µm vehicle slurry (η_{veh}).

$$\eta_{tot} = \eta_{veh}\, e^{2.7\, V_r} \quad (4)$$

where V_r = Volume Ratio of +75 µm solids in the vehicle slurry. i.e. V_r = (Volume of the “coarse”, +75 µm solids)/(Volume of the -75 µm vehicle slurry).

Therefore, having measured the plastic viscosity of the total slurry (η_{tot}), Equation 4 can be used to estimate the plastic viscosity of the -75 µm vehicle slurry (η_{veh}). In an earlier paper, Thomas (1999) found that the addition of sand to clay slimes tailings resulted in a similar ratio increase for both plastic viscosity and yield stress. On this basis Equation 4 is also used to estimate the yield stress of the -75 µm vehicle slurry from the measured total yield stress.

We now have the required information to predict the deposit velocity ($V_{d\Delta}$) using the MW&J Equations 1, 2 and 3. First, the settling velocity (W) of the representative 110 µm coarse particle size in the vehicle slurry of density ρ_{veh} and viscosity η_{veh} is calculated using established methods. In this regard, Schriek et al (1973) measured settling velocities (W) for a range of sand particle sizes and found that the measured W were essentially the same as for a sphere. This is even more likely to be the case in the turbulent pipe flow situation in which we are interested. In a quiescent fluid a non-spherical particle will align itself so as to result in the highest drag and therefore lowest settling velocity. However in a turbulent flow field the particle will present all alignments to the rapidly changing velocity field, thereby effectively giving an average equivalent settling velocity even closer to that of a sphere. Therefore on this basis, in this paper, the particle settling velocities (W) are calculated as for a sphere. With W calculated, Δ can be determined using Equation 3. F_L is then calculated using Equation 2 and $V_{d\Delta}$ predicted using Equation 1.

3. VISCOUS SUB-LAYER DEPOSITION – THOMAS (1979A)

Thomas (1979a) developed a deposit velocity prediction method for particles smaller than the viscous sub-layer, based on the Wilson sliding bed theory. Thomas argued that Equation 5 provides a lower limit to the deposit velocity. In the present context:

$$V_{d\delta}^* = 1.1\ [g\ \eta_{veh}(\rho_p - \rho_{veh})/\rho_{veh}^2\ ]^{1/3} \quad (5)$$

The $d\delta$ subscript identifies it as the viscous sub-layer deposit velocity. $V_{d\delta}^*$ is friction velocity at deposition. $V_{d\delta}^* = V_{d\delta}\sqrt{(\tau_{wveh}/\rho_{veh})}$ where τ_{wveh} = wall shear stress of the vehicle slurry at the deposit velocity.

4. APPROPRIATE VISCOSITY TO USE

The deposit velocity prediction methods given in Sections 2 and 3 involve a vehicle slurry viscosity (η_{veh}). Since the vehicle slurry is assumed to be a Bingham plastic, the question arises as to what is the appropriate viscosity to adopt. A Bingham plastic is described by the following rheological equation, and the most obvious and simplest viscosity to use is the plastic viscosity η_{pl}.

$$\tau = \tau_y + \eta_{pl}\,\gamma \tag{6}$$

However in turbulent pipe flow, η_{pl} is the relevant viscosity only at very high velocities (shear rates). As the velocity (shear rate) reduces, the effective viscosity, η_{eff} , increases above η_{pl} as per Equation 8, where τ_{wveh} is wall shear stress of the vehicle slurry.

$$\eta_{eff} = \tau_{wveh}\,\eta_{pl}\,/(\tau_{wveh} - \tau_y) \tag{7}$$

Viscous sub-layer deposition involves the thin laminar sub-layer at the pipe wall and so η_{eff} would seem the appropriate viscosity to use. The appropriate viscosity to use when calculating the particle settling velocity (W) in the MW&J prediction method is not so obvious. Given the high shear rates within eddies, η_{pl} could be considered appropriate. However the work of Rudman et al (2015) indicates that turbulent shear rates are lowest in the core region and highest in the wall region, where they are of similar order as the steady state wall shear stress rate. This suggests that η_{eff} might also be an appropriate viscosity to use in the MW&J prediction.

In comparing predictions in Section 6 with the test loop data of Goosen & Paterson (2014), predictions based on both η_{pl} and on η_{eff} have been generated. In the latter case the turbulent pressure gradient of the Bingham plastic vehicle slurry is predicted for a range of velocities using the Wilson & Thomas (1985) method, which is based on the effective viscosity. The wall shear stress is obtained from the pressure gradient and pipe diameter and equated to τ_{wveh} in Equation 7 to obtain η_{eff} at each velocity. The η_{veh} in Equation 5 is then assumed equal to η_{eff}. Similarly, η_{eff} is used to calculate the W required in Equation 3.

5. DEPOSITION COINCIDING WITH TRANSITION

Depending on the rheology of the slurry, transition to laminar flow may occur at a higher velocity than either MW&J deposition or viscous sub-layer deposition. Once laminar flow begins, there are no turbulent eddies to support the coarser particles and deposition will normally occur. The transition velocity (Vt) is largely dependent on the Bingham plastic yield stress and a number of authors have published equations for predicting Vt, with most of the following form. For example Slatter & Wasp (2000) gave Equation 8 with K=26, while Wilson and Thomas (2006) gave K=25.

$$V_t = K\,(\,\tau_y\,/\,\rho_{tot}\,)^{0.5} \tag{8}$$

6. COMPARING PREDICTED DEPOSIT VELOCITIES WITH TEST LOOP DATA OF GOOSEN & PATERSON

6.1 COMPARING TURBULENT FLOW DEPOSITION

Predictions using the above methods; MW&J (Equations 1, 2 & 3), viscous sub-layer (Equation 5), and laminar transition (Equation 8) will now be compared with the loop test results of Goosen & Paterson (2014). The author appreciates that tabulated values of concentrations and observed deposit velocities, as well as the rheograms, were provided by Goosen (2019). The Goosen & Paterson tests involved a -300 μm gold tailings with 73% -75 μm, a d_{50} of 40 μm and a solids density 2780 kg/m^3. Bingham plastic rheology parameters were determined in a rotational viscometer. The slurry was tested in loops of 100 mm, 152 mm and 242 mm internal diameter (ID) at volume concentrations (C_v) between about 12% and 47%. As noted in previous sections, in applying the MW&J deposit velocity prediction method, the slurry is split into a -75 μm vehicle slurry and a +75 μm "coarse" fraction, with the 110 μm median (d_{50}) particle size of the +75 μm fraction assumed to represent the coarse fraction.

Figures 1, 2 and 3 compare predictions with Goosen & Paterson data in 242 mm ID, 152 mm ID and 100 mm ID pipes respectively. Firstly, consider only the data points to the left of the thick transition velocity (V_t) curve, which are for deposition under turbulent flow conditions. The predicted curves between C_v=10% and 20% for all three pipe sizes, are MW&J predictions, and predict the correct behavioural trends. The full line predictions are based on η_{pl} and dashed line predictions on η_{eff}. Viscous sub-layer predictions are much lower in this concentration range and are not shown. As C_v decreases, the density and viscosity of the -75 μm vehicle slurry decrease and so provide less support to the +75 μm coarse fraction resulting in the deposit velocity increasing. The increase in the deposit velocity as C_v decreases equates to higher shear rates, meaning that η_{eff} approaches the same value as η_{pl}. Hence the two predictions become increasingly similar as C_v decreases. Given the inevitable scatter in the data, Figures 1, 2 and 3 provide no clue as to whether η_{pl} or η_{eff} is the appropriate viscosity to use in the MW&J prediction.

6.2 LAMINAR FLOW DEPOSITION

In Section 5 it was noted that once laminar flow begins, deposition will normally coincide with the transition velocity (V_t), especially in large pipe sizes. However sometimes laminar flow without deposition can occur at velocities below V_t and in some cases right down to zero velocity. It is now generally agreed that laminar flow without deposition requires a certain minimum pressure gradient. For example, Cooke (2002) quotes a private communication with Cliff Shook in 1999 who stated that a pressure gradient between 1 kPa/m and 2 kPa/m is required to transport solids in laminar flow without deposition. A minimum of 2 kPa/m was found to apply in the operation of a 5.5 km Kimberlite tailings slurry pipeline (Houman and Johnson, 2002).

We now consider the data points to the right of the transition (V_t) curve in Figures 1, 2 and 3. In none of the three Figures does deposition coincide with V_t. Consider Figure 1 (242 mm ID pipe). At C_v=31.73% transition is predicted to occur at 1 m/s but during the loop tests, deposition was not observed until 0.53 m/s. Similarly at C_v=34.90% transition is predicted at 1.35 m/s but deposition was not observed until 1.03 m/s. i.e. in both cases no deposition occurred over a significant velocity range but did eventually occur. For C_v=40.27%, no deposition was observed right down to zero velocity. Based on the measured rheology correlation for the total slurry, τ_y=14.7 Pa and η_{pl}=46 mPas at C_v=40.27%, the predicted pressure gradient for Bingham plastic laminar flow in the 242 mm pipe is around 0.3 kPa/m in the velocity range of interest. This is much less than the 1 kPa/m to 2 kPa/m minimum pressure gradient required, so deposition should coincide with V_t. The reason it does not is most likely because, in laminar flow, pipeline length is also a factor. The length of the Goosen & Paterson test loop is not stated but it is believed to be short. Particles, which may not have time to settle in a short pipe loop, may settle in a longer pipeline. This was pointed out by Thomas (1979b) and proven by the ultimate failure of the Belovo-Novosibirsk fine-coal, laminar flow pipeline in Siberia (Cowper et al, 2010). For this pipeline, tests were conducted in a 200 m long test loop with no deposition observed. However slow settling did occur along the 258 km long, 530 mm ID pipeline, which was eventually abandoned blocked up after four years operation.

Of course, if the rheology is high enough to give a pressure gradient greater than 2 kPa/m, then laminar flow without deposition down to zero velocity can occur even in a long pipeline. This would apply for the C_v=46.05% slurry in Figure 3, for which the predicted pressure gradient is 2.5 kPa/m. However no deposition also occurred for C_v=36.14% with a predicted pressure gradient of only 0.35 kPa/m suggesting the length effect is relevant.

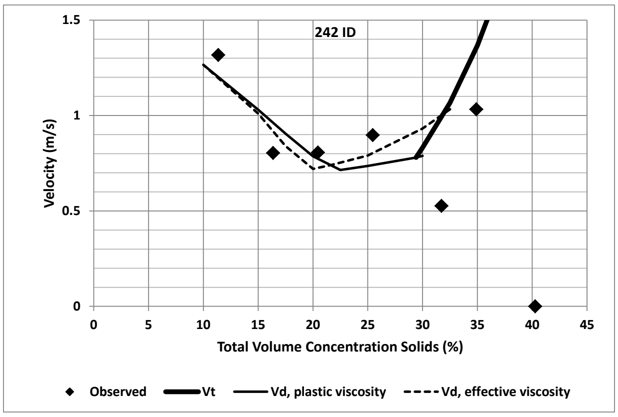

Figure 1. Predictions compared with Goosen & Paterson observations in 242 mm ID pipe

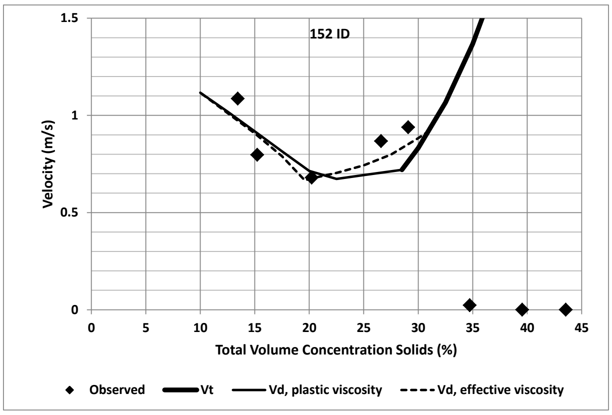

Figure 2. Predictions compared with Goosen & Paterson observations in 152 mm ID pipe

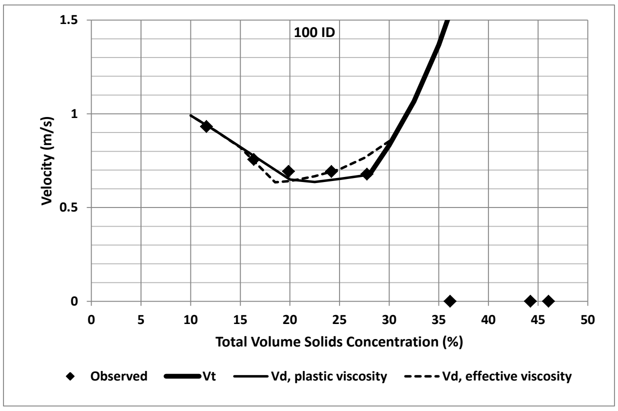

Figure 3. Predictions compared with Goosen & Paterson observations in 100 mm ID pipe

7. CONCLUSIONS

Comparisons with the test loop data of Goosen and Paterson (2014) indicate the deposit velocity for a typical fine particle, wide size distribution, Bingham plastic slurry, is well predicted using a combination of the MW&J (Thomas, 2015) method and the viscous sub-layer method of Thomas (1979a). The effective viscosity rather than the plastic viscosity appears to be most appropriate, at least for viscous sub-layer deposition.

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