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CENTRE OF CONCENTRATION IN PARTICLE-BEARING LIQUIDS

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The position of the *Centre of Concentration* $(G(r,\theta)$ or $G(x,y))$ of particle suspensions can be as important as the *centre of mass* in the mechanics of solids. Uniform suspension is indicated if it is near the geometrical centre of the duct. If it is near the periphery of the duct, sluggish or static settlement can be inferred. Particle concentration maps are now routinely obtainable, for example from tomography measurements, particle image velocimetry or enhancements to computational fluid dynamics (*CFD*) models. However when such data is not available, an approximation to the position of *G* can still be obtained using the *Two-Layer Model*. We explain how this model can be applied to the purpose. The position of *G* for a given particle concentration changes as axial velocity increases. This gives rise to a *Locus of Centre of Concentration (LCC)* which can be used to study the state of suspension of a mixture of particles in liquid. For a horizontal main the position on the locus will rise as velocity is increased until it approaches the centre of the circular section. If the locus deviates to one side or the other, circular motion at a series of velocities is indicated, but the position of *G* still approaches the geometric centre if full suspension has been achieved. The paper examines historical evidence from Electrical Resistance Tomography (ERT) in flows of beads in water and demonstrates that the *LCC* data is in reasonably good agreement with *Two-Layer Model* predictions for predominantly axial flows.

KEY WORDS: solid-liquid pipeflow, two-layer model, concentration distribution.

NOTATION

A, A_1, A_2	area of pipe cross-section, upper layer, lower layer (m^2)
Ar	Archimedes number
C_c	contact load, concentration v/v of contact layer
C_r	*in situ* concentration (over whole pipe section)
C_v	*delivered* concentration (over whole pipe section)
d	particle diameter (m)
D, R	pipe bore diameter, radius (m)
$G, G(r,\theta), G(x,y)$	centre of concentration in polar or rectilinear co-ordinates
J	polar moment of inertia ($kg\,m^2$)
s_s	relative density of particles
V, V_1, V_2, V_s	pipe velocity, velocities in upper & lower layers, velocity of solids (m/s)
x,y,z	co-ordinates: (x,y) about pipe axis, y +*ve* upwards, z +ve downstream (m)

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$(\bar{x},\bar{y}), (\overline{x_1},\overline{y_1}), (\overline{x_2},\overline{y_2})$	co-ordinates of centre of concentration, upper layer, lower layer (m)
$\alpha_1 ..... \alpha_5$	dimensionless coefficients 0.124, -0.061, 0.028, -0.431, -0.272
β	half-angle subtended by interface between layers (*radians*)
λ	hold-up ratio, *i.e.* $\frac{V-V_s}{V}$
ρ , ρ_s , ρ_m	density, solids, mixture (kg/m^3)
σ, σ_1 , σ_2	Solids loading per axial length, layer1, layer2 (kg/m)

1. INTRODUCTION

The useful *Two-Layer Model* (after Shook and Roco (1996)) bridges the gap between one-dimensional calculations using average variables in the pipe section and three-dimensional computer-intensive models. In the absence of a particle concentration distribution, a simple spreadsheet program, *2LM* (Jones, 2011), allows the calculation of the centre of concentration based on a two-layer model of the cross-section. The paper explores the plausibility of prediction using only the model. If a prediction is not possible, a good image of the particle distribution can only be obtained by experimental means, a less attractive option for the practitioner.

Given a good image of the particle distribution, one can obtain first and second moments of the contained mass about the geometrical centre of the pipe. The first moment yields the *Centre of Concentration (G)* of the particle burden. A sequence of axial velocities, or other parameters, yields a locus of points culminating in a position near the geometrical centre of the section for full suspension. Uses for the second moment, only briefly described here, exploit the apparent *solid-body* behaviour of fluid flow in a cylindrical pipe (Jones (2017)).

2. CENTRE OF CONCENTRATION USING THE TWO-LAYER MODEL

Figure 1 shows the notional 2-layer representation of the cross-section of a flowing slurry ignoring, initially at least, any circular component of the flow. The lower segment (layer 2) contains over-dense mixture with particles supported by hindered settling and the pipe walls. The upper section (layer 1) only contains particles supported by hydrodynamic forces. *2LM* is an enhanced spreadsheet version of a model of this representation (Jones (2011)) developed from a version by Shook and Roco (1996) of an original by Wilson (1976).

Particle concentrations in the two parts of the flow, C_1 and C_2, can be obtained as follows. First, a classification must be made between settling particles which contribute to Coulombic friction in a carefully defined way and those which are subject only to hydrodynamic forces. In the Two-Layer Model the assumption is made that the former group, the *contact load*, occupies the lower segment of Figure 1. The first task then is to quantify this contact load C_c.

Shook and Roco gave two "tentative" correlations for Cc, the volumetric concentration of the contact load, using dimensionless groups of variables, the first of which is used in their worked example.

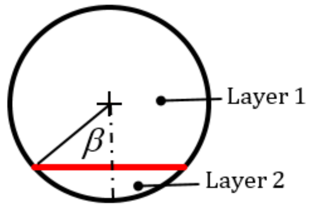

Figure 1. The two-layer model

$$\frac{C_c}{C_r} = exp\left[-\alpha_1 A_r^{\alpha_2}\left(\frac{V^2}{gd}\right)^{\alpha_3}\left(\frac{d}{D}\right)^{\alpha_4}(s_s-1)^{\alpha_5}\right] \quad (1)$$

Although this is a good fit over the whole gamut of concentrations, velocities *etc*, Equation 1 is insensitive to small changes. It provides a reasonable fit for the interface position as validated by ERT measurements but falls down on the small increments needed for the centre of concentration locus (*G)*. These small differences are magnified by problems at low subtended angles β (see Jones (2011)).

A key indicator of the veracity of the contact load is the relative delay in the solids known as hold-up (λ). Jones (2014) showed that the original version of *2LM* using Equation 1 with λ less than about 0.17 are prone to a large amount of scatter (see Figure 2).

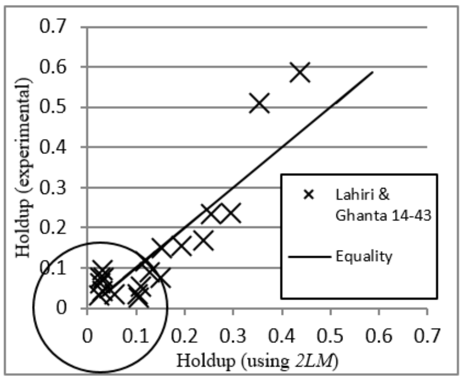

Figure 2. 2LM predictions of hold-up (λ) from Equation 1 compared to a large dataset (Jones (2014))

A compromise solution is to use Equation 1 to estimate the position of the interface and employ hold-up calculations to determine the vital contact load.

Hold-up (λ) for a flow of velocity, *V*, is defined as follows:

$$\lambda = \frac{V - V_s}{V} \qquad (2)$$

where V_s is the velocity of the solids at any section, but here considered at the delivery section. By considering the continuity of the composite volume flowrate it follows that

$$\lambda = 1 - \frac{C_v}{C_r} \qquad (3)$$

where C_v and C_r are the delivery and *in-situ* concentrations respectively. From Equation 3 the retained load in the lower section of the pipe is given by

$$C_c = C_r\lambda = C_r - C_v \qquad (4)$$

Following comments by one reviewer, the authors point out that Equation 4 does not imply that the whole of the retained load contributes to Coulombic friction. The Two-Layer Model of Shook and Roco (1996) and its spreadsheet interpretation (Jones (2011)) pay attention to the correct use of contact load in calculating Coulombic friction.

The Two-Layer Model described by Shook and Roco (1996) suggested a trial and error method to determine the hold-up. The latest version of the spreadsheet *2LM* uses an iterative procedure starting with $\lambda^{(0)} = 0.22$. Ten iterations, computing (C_v/C_r) at every stage, have been enough to secure a value consistent to at least 4 significant figures, although a smoothing function has had to be applied at very small velocities.

Equation [4] gives the contact load over the whole pipe cross-section, but in the model all of the contacting particles are in the lower segment, so the total particle loading in this segment is given by

$$\sigma_2 = \rho_s(c_c A) + \{\rho_s(c_r - c_c)A_2\} \qquad (5)$$

Note the second term in Equation 5 adds a component from the non-contact particle mixture. Strictly, the remaining particles are present throughout the cross-section. This concept is important when considering a mixture containing fine particles in suspension which pervade the intervening spaces between settled particles. In narrowly-sized mixtures this term can be ignored without significant error.

The particle loading in the upper segment is simply comprised of particles not contributing to the contact load.

$$\sigma_1 = \rho_s(c_r - c_c)A \qquad (6)$$

Now the first moments of the two particle populations can be used to obtain the Cartesian co-ordinates of the centre of concentration $G(x,y)$.

$$\rho_m c_v(\pi R^2 z)\bar{y} = \sigma_1 z\overline{y_1} + \sigma_2 z\,\overline{y_2} \qquad (7)$$

$$\bar{y} = \frac{\sigma_1\,\overline{y_1} + \sigma_2\,\overline{y_2}}{\rho_m c_v(\pi R^2)} \qquad (8)$$

The segmental areas are given by

$$A_2 = \frac{R^2}{2}[2\beta - sin(2\beta)] \;\; , \; A_1 = \frac{R^2}{2}[2(\pi - \beta) + \sin(2\beta)] \tag{9}$$

The segmental centroids are positioned as follows:

$$\left\{\overline{y_2} = \frac{-4R\,sin^3(\beta)}{3[2\beta - sin(2\beta)]}, \overline{x_2} = 0\right\} \;\; \left\{\overline{y_1} = \frac{4R\,sin^3(\beta)}{3[2(\pi-\beta)+sin(2\beta)]}, \overline{x_1} = 0\right\} \tag{10}$$

from which the position of the centre of concentration, can be obtained using Equation 8.

2.1 EXPERIMENTAL VALIDATION OF LOW CONCENTRATION DATA

We have data obtained by Electrical Resistance Tomography (see Wang, 1999) for low concentrations and low hold-up values (see Figure 3). The electrodes were placed on a horizontal leg of a pipe loop approximately 70 diameters downstream of the vertical-to-horizontal elbow with no fittings or interruptions to the flow.

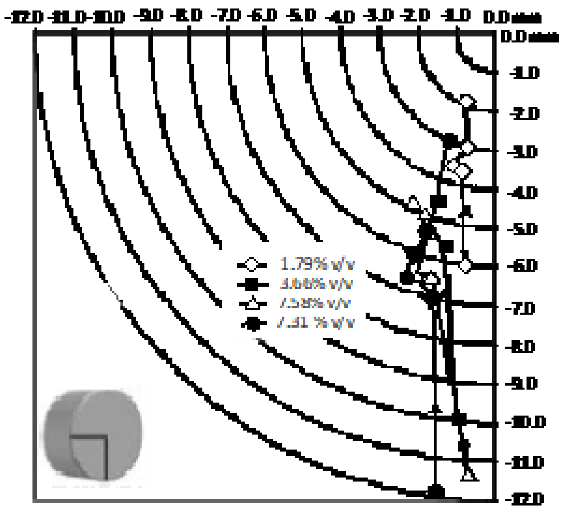

Figure 3. Centre of Concentration Loci for the lower left quadrant of a 50 mm pipe cross-section for beads (RD 1.4, diameter 1.75 mm) in water. Points were obtained for 'Nominal' axial velocities in the range {0.75, 1.0, 1.5, 2.0, 2.5 m/s} reading upwards from the lowest. After Ganeshaligam (2002)

These concentration data were used by Ganeshalingam (2002) to calculate the centre of concentration for dilute mixtures of beads in water at 'nominal' velocities of 0.75 to 2.5 m/s. Nominal fluid velocities had been inferred from pump controller settings, based on gravimetric experiments with pure water. At low values of hold-up, the position of $G(r,\theta)$ is extremely sensitive to slight changes in axial pipe velocity. Consider the data at 0.75 m/s pipe velocity. Adjustments below 0.1 m/s in the Two-Layer Model spreadsheet (2LM) bring these data onto the parity line in Figure 4.

3. DISCUSSION OF RESULTS

The results of the modelling of the concentration distribution as a two-layer model fit the experimental data reasonably well. In general the model predictions were greater (*i.e.* more negative) than the experimental results. This can be easily explained when one views the ERT diagrams. As the concentrations become more dilute, the limiting density of the lower layer becomes smaller and the interface between layers becomes more diffuse. The model tends to consign all settling particles to a segmental shape at the bottom of the cross-section even when the bed has broken down into a roughly circular diffuse area. This puts the centre of concentration, $G(r,\theta)$, at a lower point than it should be. The data was dilute and these effects would be much diminished with denser mixtures.

The leftward kink in all loci of experimental data between 1.5 and 2.0 m/s is particularly interesting. It indicates an adventitious clockwise circulation around the body of settled particles as it lifts from the pipe bottom.

Mention has been made of the influential variable *hold-up* (λ). Table 1 shows the hold-up estimates we have now achieved. For the majority of the current predictions the hold-up falls below 0.17, but this no longer causes the difficulties shown in Figure 1.

Table 1

Estimates of Hold-up

Concentration		**Nominal axial velocity**				
Overall	**Lower Layer**	**0.75**	**1.0**	**1.5**	**2.0**	**2.5**
%	**%**	**m/s**	**m/s**	**m/s**	**m/s**	**m/s**
1.79	20	0.1990	0.1651	0.1373	0.1261	0.1203
3.66	40	0.2616	0.2045	0.1568	0.1374	0.1275
5.48	50	0.2945	0.2251	0.1657	0.1412	0.1286
7.31	50	0.3002	0.2280	0.1642	0.1373	0.1234

In high concentration pastes and slurries with non-Newtonian rheologies the *2LM* representation may be less applicable because the lower layer may be in a laminar regime or may be below the yield point (see Pullum *et al* (2004)). In these cases the Locus of Centre of Concentration will provide an invaluable insight into the flow. With this in mind, the authors are gathering concentration profile data for complex mixtures in order to apply the *LCC* technique directly or via an enhanced model.

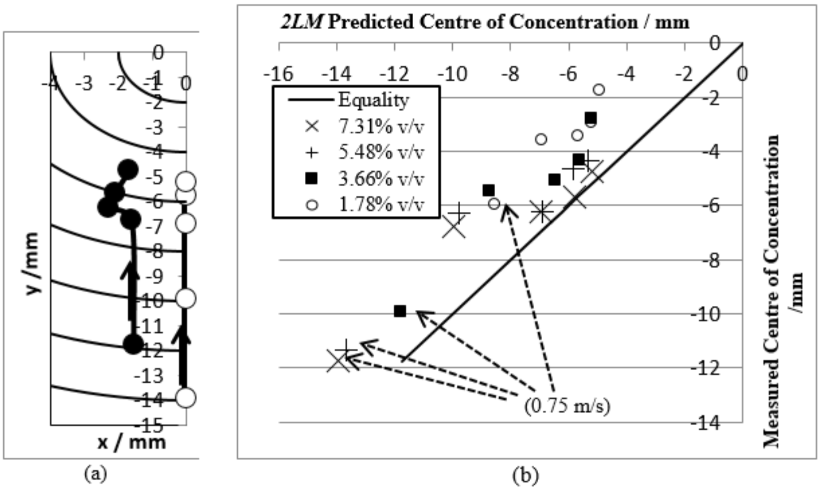

Figure 4. (a) 7.31% data from Figure 2 illustrated as black circle markers. The points are for nominal pipe velocities 0.75 m/s to 2.5 m/s, reading upwards from the lowest. Predicted outcomes using the Two-Layer Model are illustrated with white circles. (b) Correlation between the measured and predicted centre of concentration positions.

4. RECOMMENDATIONS

The experimental data described in §2.1 and shown in Figures 2 and 3 was measured 17 years before being published here. It should be repeated and augmented with results from denser mixtures. Luckily the data was ideally suited to explore the low concentration efficacy of the Two-layer Model in establishing the centre of concentration of a flowing slurry. We now need to apply the technique more widely to mixtures which do not necessarily lend themselves to *2LM* representation. This is an avenue for further work and a later paper.

The Two-layer Model is not the only means to quantify the centre of concentration loci in the absence of experimental data. Computational Fluid Dynamics (CFD) models can incorporate particle models.

We have presented *LCC* loci for the variation of pipe velocity but the technique can also be applied to variations in concentration or other variable at given velocities. For example the position of the particle burden has a great bearing on the wear propensity of a piping systemand the authors believe that a new study of wear at different values of centre of concentration $G(r,\theta)$ would yield positive benefits. Following input from a reviewer the authors point out that the locus of centre of concentration (with corresponding wear data) will indicate a trajectory for remedial measures. For example the effect of pipe velocity or concentration on wear can be inferred from interpolation or extrapolation of this trajectory.

Away from duct walls, a simple liquid acts as a solid body in axial and circumferential motion (see Jones (2017)). In circular motion the liquid cylinder can be represented as a fluid flywheel with viscous friction. This representation yields a first-order system in which the polar moment of Inertia, *J*, is proportional to the time constant. For flow including settling particles we propose second-moment determinations for calculation of *J*, and from this time constants and energy required to lift particles into circular motion: a cue for more research and another paper.

5. CONCLUSIONS

The *Centre of Concentration* of a particle-bearing liquid has been shown to be a powerful concept. When presented as a locus (*LCC*) with changing velocity, concentration or other property, it has been shown to demonstrate the changing behaviour of the mixture in suspension and propensity to wear.

We have applied the *Two-Layer Model* to construct an approximation to an *LCC* and demonstrated a reasonable correlation to the locus obtained from actual data. The model slightly over-estimated downward displacements of the centre of concentration $G(x,y)$ as a consequence of the diffuse interface in the real situation, but the overall performance was very encouraging. It has to be expected that in certain situations the two-layer representation will be less successful than the case we have described. The application of the *LCC* technique to actual data will always be preferable.

Experimental loci provide good indications of circulation and turbulence as velocities approach the deposition limit. Ganeshalingam (2002) has shown the effect of swirling flows on the centre of concentration loci. A clockwise circulation causes the loci to swing to the left, but the movement towards the centre is still an indication that the flow approaches fully suspended status. Work with an historical data set shows a slight circulation effect as settled mass starts to become detached from the bottom of a pipe.

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