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KINETIC THEORY BASED APPROACH TO MODELLING OF SEGREGATION IN INTENSE BIMODAL BED LOAD TRANSPORT

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Intense bed load grains are transported over an erodible bed through a transport layer. The local concentration of grains varies considerably across the transport layer from zero at the top of the layer to values near the bed concentration at the bottom of the layer. Kinetic theory (KT) offers constitutive relations which relate local granular stresses with local concentration and velocity (gradient) in collision-dominated granular flows. Extended kinetic theory takes care of flows at high concentrations where clustering of grains dominates over binary collisions.
The goal of the present work is to test the kinetic theory based approach by Larcher and Jenkins to modelling of a segregation process in intense bimodal bed load at conditions observed in our tilting-flume experiments. For the bimodal flows, we observed a development of a sliding layer of finer grains at the top of the deposit as a result of the segregation process. We show that the development of such a layer is also a result of the kinetic-theory based model simulating segregation of two fractions which differ in grain size.

KEY WORDS: sediment, concentration distribution, extended kinetic theory, particle collision

1. INTRODUCTION

Particle segregation takes place in many industrial and natural processes in which transport of solid particles is involved. Sorting is associated with relative movement of particles of different fractions in the moving mass. The movement can be seen as flow caused by shearing of the mass. Segregation as a product of intergranular collisions has direct impact on a behavior of the granular flow. For instance, segregation is typical for dry gravity-driven flow recognized as “the falling of snow and rock avalanches” on steep hillsides of mountains. A main force acting on transported particles, apart from the gravitational force, is the force generated by particle collisions. We can look at particle collisions from two different perspectives. One perspective follows movement of each individual particle in flow. The other considers moving particles as a continuum with properties determined on a basis of average characteristics of the movement of the individual particles.

Kinetic theory (KT) of granular flow is based on statistical methods developed for gases. KT-based analyses of solid-liquid flows governed by collisions of solid particles have been made by a number of authors, Armanini et al. (2005) or Berzi and Fraccarollo (2013) among others. Most of the analyses focused to collisions of mono-size grains. Works on collisional interactions among particles of different sizes have been scarce (e.g. Jenkins and Mancini 1989) and focused primarily to dry flows. Larcher and Jenkins (2013) proposed a segregation theory and developed a KT-based model for final segregation of binary mixtures. They extended it later to modelling of time development of the segregation in Larcher and Jenkins (2015) and validated the segregation model for dry granular flow.

Not only segregation of grain fractions in dry flows is important, the particle segregation process in flowing water is of importance too, imagine e.g. intense transport of bed load during flash floods on mountain streams. In our previous work, we looked at principles of intense bed load of narrow graded fractions of sediment in the upper plane bed regime (e.g. Matoušek et al. 2015). Recently, we extended our investigation to intense transport of bimodal mixtures (Zrostlík et al. 2016). One of the observed interesting phenomena was a development of a sliding layer composed by finer fraction at the interface between the collisional layer and bed deposit in the bimodal flow (Fig. 1). In our 2016-paper, we analyzed the flow at steady state after particle segregation was finished (including the final development of the sliding layer). In this paper, we analyze and simulate a process of evolution of particle segregation in bimodal flow.

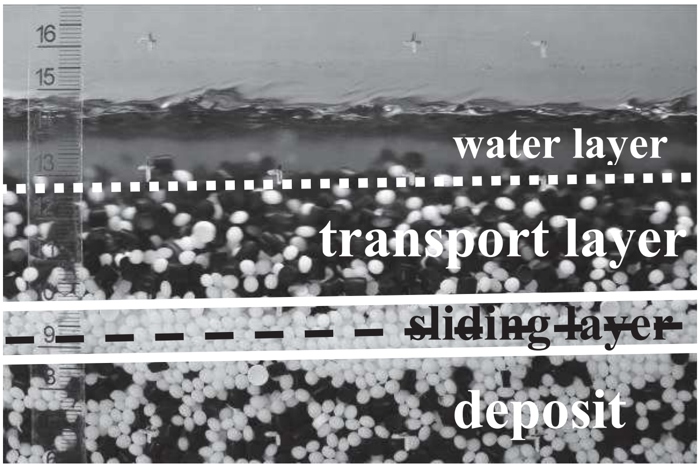

Fig .1 Distribution of grains of sediment fractions in bimodal flow in upper plane bed regime (black – coarser grains, white – finer grains). Legend: white solid lines – boundaries of interfacial sliding layer, black dashed line – top of stationary deposit, white dashed line – top of collisional transport layer

2. TIME SEGREGATION

We employ the segregation model presented by Larcher and Jenkins (2015). It is based on the extended kinetic theory and considers dry flow of two fractions (*A,B*) of spherical grains of not too different size (radius *r*) and density (mass *m*). The authors validated their model for quite steep flows of high concentration of transported particles. In the model, the evolution of segregation is expressed for spatial positions (*x,y*) and time of segregation (*t*) expressed using the dimensionless time (τ). The theory assumes uniform distribution of grains, i.e. a constant concentration *c* across the flow depth, so that the segregation affects only the distribution of individual fractions (the local concentrations of fraction c_A and c_B) and $c = c_A + c_B$ = const. For the sake of segregation evaluation, the concentration is expressed as the number density n_A, n_B ($c_A = 4\pi n_A r_A^3/3$). The measure of segregation *X* is defined as $X \equiv (n_A - n_B)/2n$, where $n = n_A + n_B$. As for the steady segregation (Larcher and Jenkins 2013), the quantity of segregation *X* is transformed to a dimensionless parameter ζ through relation $X = \frac{(\hat{c}_A + \hat{c}_B)}{2c}\zeta$ where $\hat{c}_A \equiv \frac{\overline{n}}{\overline{n}_A}\overline{c}_A$ and the overbar-sign notes an average over the flow depth. The following KT-based parameters are dependent on segregation and distribution of collisions: Γ_1, Γ_2, R_1, R_2, G (Larcher and Jenkins 2013). For more details about the parameters used in the model, see Larcher and Jenkins (2015). In the segregation model, Equation 1 solves temporal evolution of vertical segregation at one location,

$$\frac{\delta\zeta}{\delta\tau} = \left(\frac{r_A + r_B}{H}\right)^{3/2} \frac{(\pi\cos\phi)^{1/2}}{128G^{3/2}} \left(\frac{2}{1+e}\right)^{1/2} \frac{2c}{(\hat{c}_A + \hat{c}_B)} \times$$
$$\times \frac{\delta}{\delta z}\left\{ \frac{\left[(2(1+e)G\Gamma_2 - \Gamma_1)\delta m + (2(1+e)GR_2 - R_1)\delta r\right]}{(1-z)^{1/2}} \right.$$
$$\left. \times\left[1 - \frac{(\hat{c}_A - \hat{c}_B)}{c^2}\zeta^2\right] + 2(1-z)^{1/2}\frac{(\hat{c}_A - \hat{c}_B)}{c}\frac{\delta\zeta}{\delta z}\right\} \tag{1}$$

in which *H* is the flow depth, *z* is the dimensionless vertical position (the dimensional vertical position *h* above the flow bottom is normalized by *H, z=h/H)*, Φ is the longitudinal slope of flow, *e* is the coefficient of restitution of colliding particles.

Boundary conditions must be defined to solve the evolution using Eq. 1. Those are expressed for the vertical flux of particles computed by the term in the complex brackets in Eq. 1. The flux must be zero at the bottom of the flow because no particles are in motion at this boundary. The flux must be zero at the top of the flow as well because no particles are present.

The initial condition is given by a known distribution of individual particles at the beginning of the segregation process. A ratio fraction volumes of A and B is determined at the initiation of the process and perfect mixing is assumed.

After solving the parabolic-elliptic partial differential equation (Eq. 1), we determine concentrations of both fractions from the quantity of segregation (X) in time and from the definition of segregation as

$$c_A = -\frac{(2X+1)c}{(2X-1)-(2X+1)(r_B/r_A)^3} \tag{2a}$$

$$c_B = \frac{(2X-1)c}{(2X-1)-(2X+1)(r_A/r_B)^3} \tag{2b}$$

An example of one set of computational results is plotted in Figure 2. The results are obtained for flow conditions similar to those handled in Larcher and Jenkins (2015). For the modelled dry flow, we consider particles of the same density, flow inclination $\Phi = 25°$, relation of radiuses $r_B/r_A = 1.1$ and coefficient of restitution $e = 0.65$. The chosen value of e is typical for glass beads colliding with an obtuse impact angle (Salman et al. 1991). The left plot, (a), shows the time evolution of the dimensionless segregation and the right plot, (b), shows the time evolution of concentration profiles of both fractions.

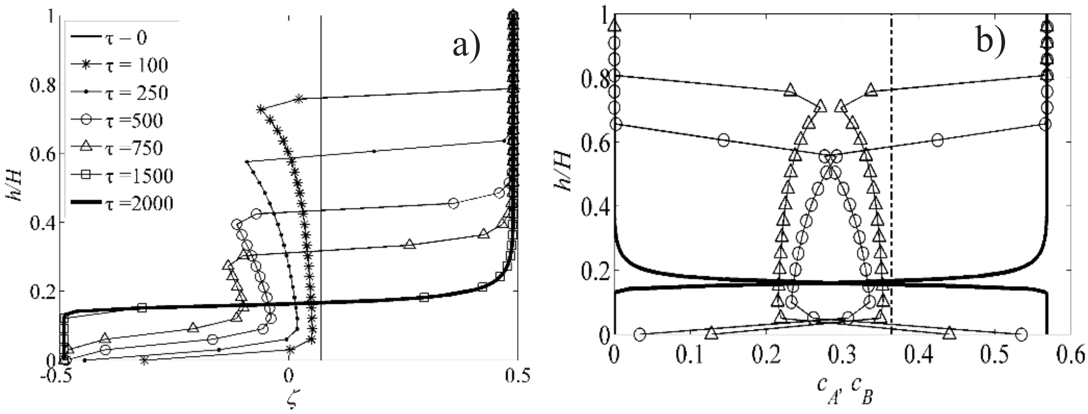

Fig .2 a) Time evolution of dimensionless segregation in uniform concentration profile, b) Time evolution of concentration profile of both fractions due to sorting. Legend: dash dot - initial state ($\tau = 0$), triangle - situation at $\tau = 500$, circle $\tau = 1000$, thick line $\tau = 2000$

The plots indicate how the process of segregation works. Due to collisional interaction among particles the two fractions tend to completely separate from each other and the finer fraction migrates down while the coarser fraction migrates up. At certain $\tau > 0$ during the segregation process, the flow depth is divided into 3 zones:

- in the upper zone the segregation process is completed, the zone contains only coarser particles and the distribution of grains is uniform;
- the segregation process is going on in the central mixed zone, where both fractions exhibit steep concentration gradients;
- in the lower zone the segregation process is completed, the zone contains only finer particles and the distribution of grains is uniform.

For the flow conditions as in Fig. 2, we tested sensitivity of model predictions for the time evolution and final state of segregation on the coefficient of restitution *e*, which is defined as the ratio of the post-collision- to pre-collision velocity difference of two colliding particles. Values of the coefficient may vary from 0 (inelastic collisions) to 1 (perfectly elastic collisions). Figure 3 shows the effect of a value of the coefficient of restitution on the dimensionless sorting at two time steps: $\tau = 250$ and $\tau = 2000$. The plots demonstrate that more elastic collisions accelerate the segregation and the highest value of *e* does allow the entire segregation of the fractions although the process is completed at $\tau = 2000$.

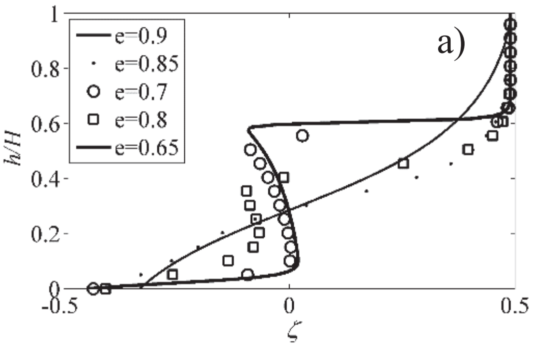

Fig .3 Sensitivity of dimensionless segregation on coefficient of restitution, a) situation at τ = 250 (-), b) τ = 2000 (-)

As a next step, we carry out the same simulation as for Fig. 2 but this time for plastic particles (we used plastic grains in our experiment described in Zrostlík et al. 2016), hence we choose e = 0.8 for the simulation. At the moment, our selected value of *e* is just a guess. The time evolution and the final distribution of particles are showed in Figure 4. A comparison of the simulation results in Figs. 2 and 4 again demonstrates that the segregation is faster for grains with more elastic collisions, i.e. our plastic grains instead of beads used in Larcher and Jenkins (2015).

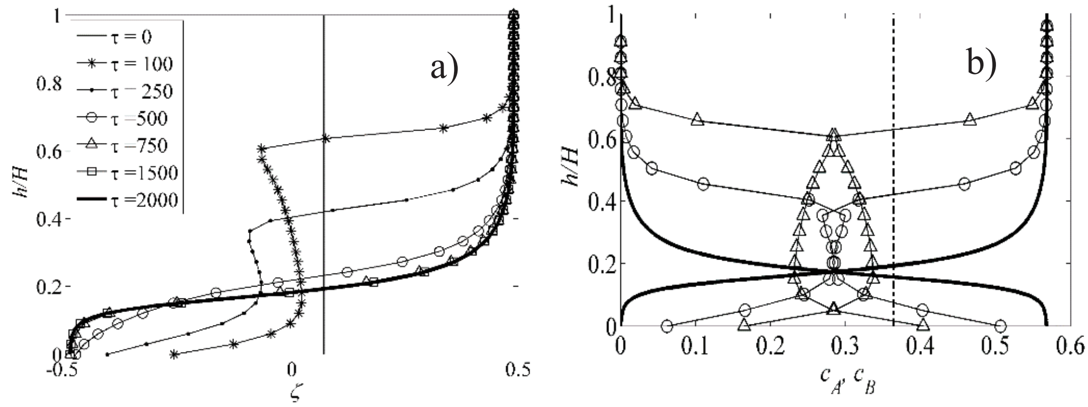

Fig .4 Time evolution of concentration profiles of both fractions for rB = 1.1rA and e = 0.8 (typical for plastic grains), Legend: dash dot - initial state (τ = 0), triangle - situation at τ = 500, circle τ = 1000, thick line τ = 2000

3. MODIFICATION OF SEGREGATION FOR LINEAR PROFILE

A concentration profile in bed load transport is not uniform. Instead, it is linear across the collisional transport layer through which particles are transported in the upper plane bed regime (Matoušek et al. 2015). For modelling of the segregation process in bimodal bed load, we focus to the collisional transport layer in which the segregation takes place. We assume the zero concentration at the top of the collisional layer and the maximum local concentration of 0.586 (which corresponds with the theory by Larcher and Jenkins 2015) at the bottom of the layer.

For a computation of the segregation model (Eqs. 1-2) it means that we must replace the originally assumed uniform profile of $c(z) = 0.586$ with the linear profile of $c(z) = (c - z \cdot c)$. A height-variable local concentration makes also other c-related parameters of the segregation model variable with height within the collisional transport layer. Unfortunately, it appeared that his height variability made the model computationally quite unstable. For the sakes of model stability, we decided to apply the height variation only to the local concentration c and to the dimensionless segregation X (and hence ζ). The other parameters of the segregation model (Γ_1, Γ_2, R_1, R_2, G) used a value of the flow-depth averaged concentration at the initial condition instead of c.

Another mathematical complication was associated with values of local concentrations at the collisional layer boundaries, leading to dividing by zero in the model equations. To avoid this problem, computations were carried out only in the range of 0.1 to 0.9 of the dimensionless vertical position within the layer. The boundary conditions for the computation again defined that the granular fluxes were zero at the top and at the bottom of the collisional layer. The initial conditions were the constant ratio of volumes of both fractions expressed as $c_A / c = 0.5$ in the layer and perfect mixing of fractions at the initiation of the segregation process.

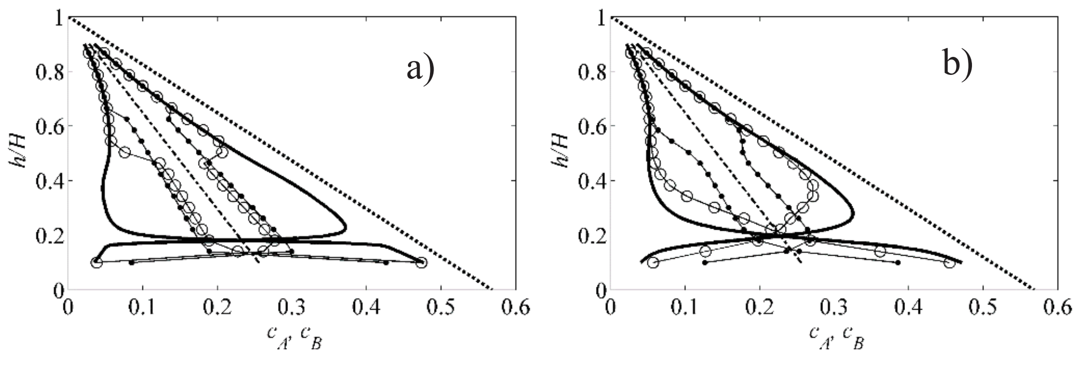

Fig .5 Time evolution of originally linear concentration profiles of both fractions for $r_B = 1.1 r_A$ and a) e = 0.65, b) e = 0.80; dash dot - initial state ($\tau = 0$), line with point- situation at $\tau = 100$, line with circle $\tau = 250$, thick solid $\tau = 2000$

Figure 5 shows the time evolution of shapes of concentration profiles of both fractions from a linear profile at the initiation of the segregation process to the final state after the segregation process was finished. The left plot (a) gives a solution for grains of the coefficient of restitution e = 0.65. The right plot (b) gives the same for e = 0.8. Both plots show a development of a layer composed of finer particles above the top of the deposit. This corresponds with the development of the sliding layer composed of finer particles and observed in our experiments with bimodal mixtures (Fig. 1). Note also that the segregation with e = 0.65 produces a sharper interface between the segregated layers of coarser grains and finer grains.

In Figure 6, we demonstrate how long is the period of time over which segregation evolution takes place before it reaches the final equilibrium. The time evolution is demonstrated on a change of a vertical position within the collisional layer of the center of mass of an area occupied by concentration profiles of individual fractions. IN the segregation process the center of mass of coarser grains moves up and the center of finer grains moves down. The left plot is a solution for a uniform distribution of total concentration, while the right plot is a solution for a linear distribution of total concentration. The segregation process is faster and the new equilibrium is found earlier in flow with the linear concentration distribution than in flow with the uniform distribution. This is due to the fact that less particles are present in the upper part of the collisional layer than in the lower part if the profile is linear at initiation of the segregation process and therefore the process takes less time to finish.

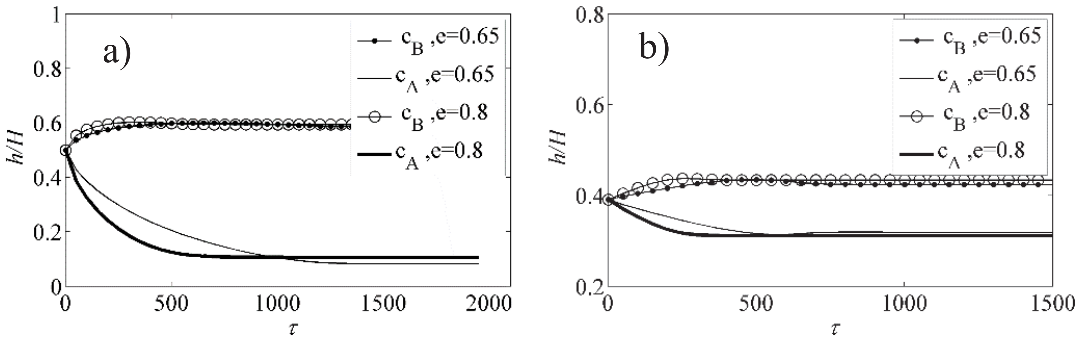

Fig .6 Time evolution of vertical position of center of mass of area occupied by concentration profile for two fractions and different values of coefficient of restitution for simulation using a) uniform concentration distribution, b) linear concentration distribution

4. CONCLUSIONS

We employed a kinetic-theory based model to study segregation of fractions in bimodal bed load transported through a collisional transport layer in the upper plane bed regime. We employed the Larcher-Jenkins segregation model proposed for highly concentrated dry flow with uniform distribution of spherical beads.

Our interest is in collisional flows of plastic grains with linear concentration profiles. A variation of the coefficient of restitution in the model reveals that the simulated segregation process is faster for plastic grains with a higher value of the coefficient than for the glass beads used by Larcher and Jenkins.

We modify the model for collisional bed load with a linear concentration profile. For given assumptions, the modified segregation model predicts a presence of a high concentrated layer of finer grains at the interface between the deposit and the collisional transport layer. The presence of such a layer was detected in our earlier bimodal experiments in a laboratory tilting flume.

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REFERENCES

1. A. Armanini, H. Capart, L Fraccarollo and M. Larcher, 2005, Rheological stratification in experimental free-surface flow of granular-liquid mixtures, Journal of Fluid Mechanics, 532
2. D. Berzi and L. Fraccarollo, 2013, Inclined, collisional sediment transport, Physics of Fluids, 25, 106601
3. J.T. Jenkins and F. Mancini, 1989, Kinetic theory for binary mixtures of smooth, nearly elastic spheres, Physics of Fluids A 1 (12)
4. M. Larcher and J.T. Jenkins, 2013, Segregation and mixture profiles in dense, inclined flows of two types of sphere, Physics of Fluids 25, 113301
5. M. Larcher and J.T. Jenkins, 2015, The evaluation of segregation in dense inclined flows of binary mixtures of spheres, Journal of Fluid Mechanics, 782
6. V. Matoušek, V. Bareš, J. Krupička, T. Picek and Š. Zrostlík, 2015, Experimental investigation of internal structure of open-flow with intense transport of sediment, Journal of Hydrology and Hydromechanics, 63, 4
7. A.D. Salman, A Verba, Z. Lukenics and M. Szabó, 1991, Effects of impact velocity and angle on collision, Periodica Polytechnica Chemical Engineering, 35, 1-2
8. Š. Zrostlík and V. Matoušek, 2016, On the effect of segregation on intense bimodal bed load, EPJ Web Conf. Vol. 143, 2017 (EFM16 – Experimental Fluid Mechanics 2016)
9. . Matoušek, V. Bareš, J. Krupička, T. Picek and Š. Zrostlík, 2015, Experimental investigation of internal structure of open-flow with intense transport of sediment, Journal of Hydrology and Hydromechanics, 63, 4
10. A.D. Salman, A Verba, Z. Lukenics and M. Szabó, 1991, Effects of impact velocity and angle on collision, Periodica Polytechnica Chemical Engineering, 35, 1-2
11. Š. Zrostlík and V. Matoušek, 2016, On the effect of segregation on intense bimodal bed load, EPJ Web Conf. Vol. 143, 2017 (EFM16 – Experimental Fluid Mechanics 2016)