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CHANNEL WIDTH CALCULATION FOR TAILINGS BEACHING

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Self-formed channel profiles on tailings beaches are determined by rheology, settling solids and flow conditions. A model is described for the width of subaereal tailings channels with settling solids flowing at onset of turbulence. Early transition of open channel with viscous Non-Newtonians is taken into account building on the homogenous slurry open channel data set of Haldenwang. Free formed channels measured in on-site tilting flume tests by Pirouz et al. are analysed. It is confirmed that a number of these channels is at onset of transition and that their width/depth ratio is reasonably well predicted. Other channels in this data set are at low Re laminar flow at minimal width-depth ratio, settling solids apparently being discharged.

KEY WORDS: Laminar flow, onset transition, revisit Haldenwang data set, open channel.

1. INTRODUCTION

Channel formation is observed in tailings beaching. The prediction of channel width is an intricate and unsolved problem. For given discharge, channel width determines in-channel flow velocities and these need to be known in order have any chance of predicting the morphological development of beaches. Channel formation of shear thinning slurries is evaluated to provide reference material for numerical developments in Delft3D, Sittoni et al. (2016). The flow can be laminar, turbulent or in between. Persistent tailings channels have been observed over substantial time. These channels are transporting the tailings to flatter area where they deposit, whereas in-channel sedimentation is marginal, Fitton et al. (2008) and Williams (2014). Therefore they flow at a near equilibrium of settling and (re)suspension. For the concerning rheology, the settling of solids is slow and the current working hypothesis is that the slightest turbulence prevents deposition. Haldenwang (2003) provides a useable empirical relation for the onset of the laminar-turbulent transition based on open channel tests with homogenous shear thinning fluids. This onset can occur for lower Reynolds numbers than for Newtonian fluids. The data-set is revisited to see if there is a relation with width-depth ratio which is important to our understanding, Section 3.

Following first principles, these channel flows have to satisfy continuity, momentum balance and the mentioned onset condition. Model calculations are compared in Section 4 with self-formed channel conditions measured in on-site tilting flume tests reported by Pirouz et al. (2013).

The analysis applies only to channels flowing through an existing deposit. If tailings are flowing over an existing topography instead, lubrication flow applies, with wider lateral extent, Coussot (1997).

2. THEORY

Three physical criteria; continuity, mechanical equilibrium and onset of transition have to be satisfied. In the current approach specific assumptions are:

1) no slip: no slip planes or shear banding as observed in wide flume tests, Van Kesteren et al. (2015),

2) the banks withstand fluid shear stresses strengthened by either deposited solids (Talmon, 2010), thixotropy or dewatering (aging),

3) the channel cross-section is rectangular.

Flow rate Q and energy slope i (= channel slope) are given.

In the proposed methodology, the flow conditions are evaluated for a variation of cross-sectional averaged wall shear stress (τ) and width/depth ratio (W/h). A solution is found when outcomes of all three physical criteria, currently expressed as Re_τ (equivalent to friction factor f=64/Re_τ), converge.

<u>Criterion #1 continuity</u>:

Starting with the definition of Re_τ and expressing the mean velocity U by means of continuity in the flow rate Q of rectangular open channel flow gives:

$$Re_\tau = \frac{8\rho U^2}{\tau} = \\ = \frac{8\rho Q^2}{\tau}\frac{1}{h^4}\left(\frac{h}{W}\right)^2 = \frac{8\rho Q^2}{\tau}\left(\frac{i\rho g}{\tau}\right)^4\left(\frac{1}{\sqrt{W/h}+2/\sqrt{W/h}}\right)^4 \qquad (1)$$

where ρ= mixture density, g= gravity, h=flow depth, W=channel width. Note that the highest Reynolds number (or lowest friction factor) is attained at W/h=2. The average wall shear stress τ is related to the hydraulic radius (R_h) by:

$$\tau = i\rho g R_h \quad , \quad R_h = Wh/(W+2h) \text{ and } \tau = \tau_y + K\left(\overset{*}{\gamma}\right)^n \qquad (2a,b,c)$$

where τ_y= yield stress, K= viscosity index, n= flow index, γ^* = shear rate. Given Q and i, there are three unknown: Reynolds number Re_τ , wall shear stress τ and width/depth ratio (W/h).

Criterion #2 mechanical equilibrium:
Two different asymptotic theoretical approaches are available for internal mechanical equilibrium: a pipe flow analogy (representing a half open pipe conduit shape) or an infinite wide open channel analogy. For an infinite wide open channel the mathematical expression for bulk shear rate is:

$$\frac{3U}{h}=\frac{3U}{R_h}=\frac{3n}{2n+1}\left(\frac{\tau-\tau_y}{K}\right)^{1/n}\left(1-\frac{\tau_y}{\tau}\right)\left(1+\frac{n}{n+1}\frac{\tau_y}{\tau}\right) \tag{3}$$

and if the Reynolds number, see Eq.(1), is reformulated with Eq.(2a) into:

$$Re_\tau=\frac{8}{9}\frac{\rho\tau}{(i\rho g)^2}\left(\frac{3U}{R_h}\right)^2 \tag{4}$$

the Reynolds number is expressed in rheological properties, wall shear stress and energy slope. For pipe flow the bulk shear rate and Reynolds number are:

$$\frac{8U}{D}=\frac{2U}{R_h}=\left(\frac{\tau-\tau_y}{K}\right)^{1/n}\left(1-\frac{\tau_y}{\tau}\right) \left(\frac{4}{1+3n}\left(1-\frac{\tau_y}{\tau}\right)^2+\frac{8}{1+2n}\left(\frac{\tau_y}{\tau}\right)\left(1-\frac{\tau_y}{\tau}\right)+\frac{2}{n+1}\left(\frac{\tau_y}{\tau}\right)^2\right) \tag{5}$$

$$Re_\tau=\frac{8}{4}\frac{\rho\tau}{(i\rho g)^2}\left(\frac{2U}{R_h}\right)^2 \tag{6}$$

Note that for Newtonian fluids these two Reynolds numbers differ by a factor 9/4 for the same wall shear stress, energy slope, density and viscosity. In case of non-Newtonian fluids their difference can become substantially larger.
These two approaches are weighted on basis of the W/h ratio utilising calculations of Straub (1958) for friction factor of laminar Newtonian fluid flow in ducts, which because of symmetry is here applied to open channels, Figure 1.
The friction factor is f=64/Re_{4Rh} for pipes having a bulk shear rate of $2U/R_h$, and is f=96/Re_{4Rh} for wide open channels which have a bulk shear rate of $3U/R_h$. Definitions are:

$$Re_{Rh}=\frac{\rho U 4R_h}{K} \quad , \quad f=\frac{\mathrm{M}}{Re_{Rh}} \quad , \quad \text{bulk shear rate: } \frac{\mathrm{x}U}{R_h} \tag{7}$$

Where M and x are constants which can be read from Figure 1. This figure is created from a mathematical formulation provided by Straub. The applied weighing methodology for W/h>4.5 is: Re_τ= (x-2)*Eq.(4)+ (3-x)*Eq.(6). For smaller W/h only Eq.(6) applies, describing pipe flow.

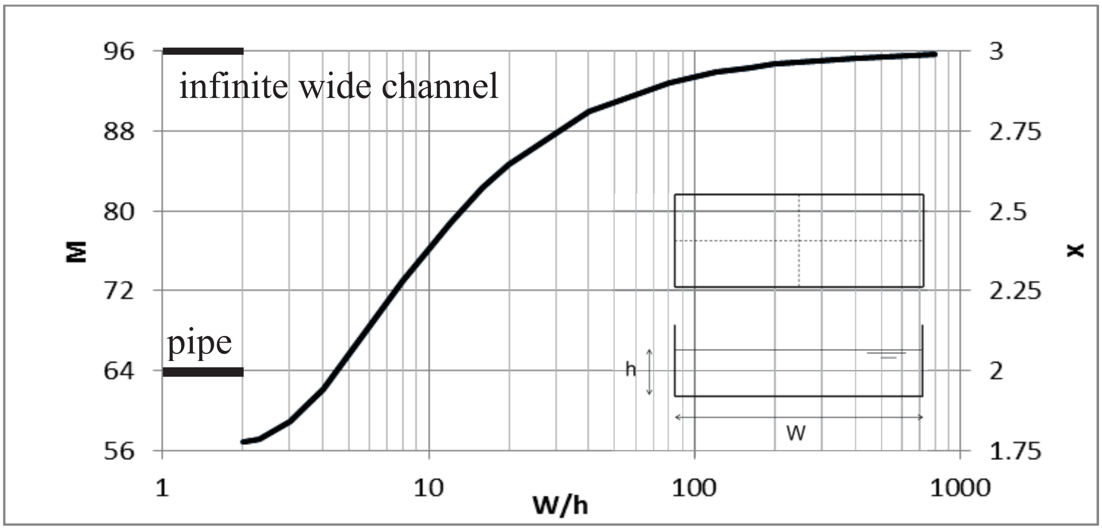

Fig.1 Straub's graph for M-factor in friction factor model and x-factor in bulk shear rate

<u>Criterion #3 onset transition criterion</u>:
Haldenwang's empirical criterion for onset of transition is:

$$Re_{2ypp} = \frac{200}{\mu_{a,100}^{0.21}} Fr + \frac{71}{\mu_{a,100}^{0.75}} \tag{8}$$

Definitions for yield plastic Reynolds number (Re_{2ypp}) and Froude (Fr) number are:

$$Re_{2ypp} = \frac{8\rho U^2}{\tau_y + K\left(2U / R_h\right)^n} \quad , \quad Fr = \frac{U}{\sqrt{gh}} \text{ and} \tag{9ab}$$

$$\mu_{a,100} = \frac{\tau_y + K\left(100\right)^n}{100} \tag{10}$$

where $\mu_{a,100}$ = apparent viscosity at a shear rate of 100 1/s.
The denominator in Eq.(9a) is an approximation for bed shear stress.
Re_{2ypp} and Fr can both be analytically expressed in Re_τ:

$$Re_{2ypp} = \frac{\tau / \tau_y}{1 + \frac{K}{\tau_y}\left(\sqrt{\frac{Re_\tau \rho}{2\tau}} ig\right)^n} Re_\tau \quad , \quad Fr = \sqrt{\frac{i}{8}\frac{1}{1+2h/W}}\sqrt{Re_\tau} \tag{11}$$

Re_τ at onset of flow transition is solved by iteration of Eq.(8) and Eq.(11).

An overview of constant parameters, varying parameters and outcome is given in the Table 1. A solution situated at onset of transition is obtained when the calculated Re_τ of all three criteria is the same.

Table 1

Overview parameters in a calculation

criterion	i	ρ	Q	τ_y	K	n	var W/h	var τ	outcome
#1	x	x	x				x	x	Re_τ
#2	x	x		x	x	x	x	x	Re_τ
#3	x	x		x	x	x	x	x	Re_τ

3. REVISIT OF HALDENWANG OPEN-CHANNEL DATA

Transition data had to be retrieved. The applied procedure to determine onset conditions is a bit different from Haldenwang's original. For each fluid and channel width, the results are plotted as a pseudo-rheogram (Fitton and Slatter 2013 approach): wall shear stress as a function of bulk shear rate. This makes the laminar flow data to coincide, onset of transition is identified when the first data point rises above the laminar flow curve (for low-sloping channels this produces earlier onset as in Haldenwang's approach). It is observed that the transition Re-number is systematical lower for low *W/h* ratio, Figure 2. This could indicate that channel geometry might play a role in early onset, because of a complicated internal distribution of shear rate and shear stress.

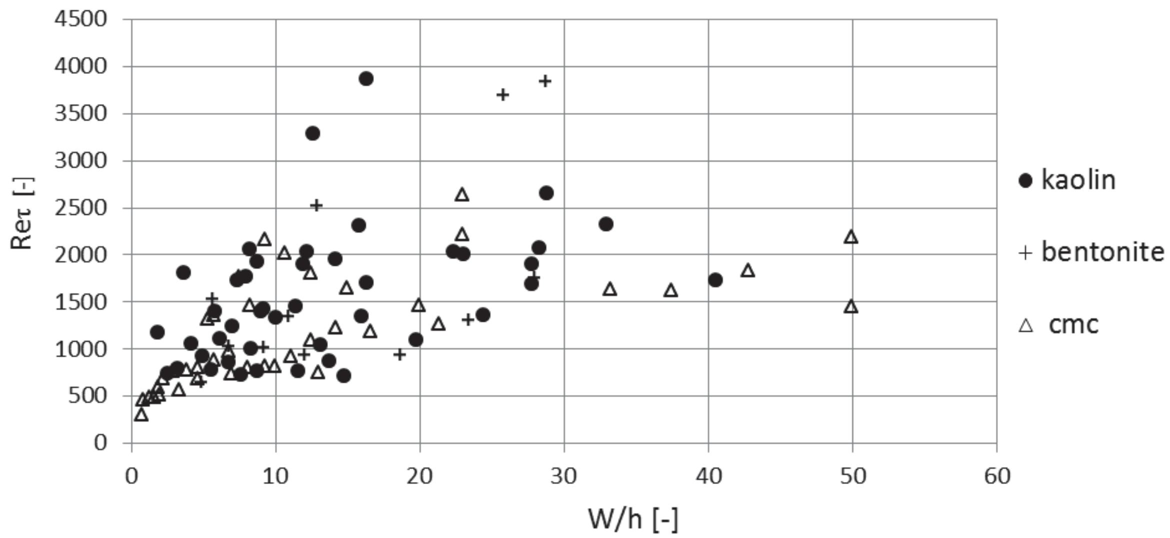

Fig.2 Width/depth ratio of channels in Haldenwang data-set and Re_τ at onset of transition

4. THEORETICAL CHANNEL WIDTH PREDICTION

Conditions of self-formed channels in on-site flume tests, as described by Pirouz et al. (2013), are taken for evaluation of predictive capabilities of the theory. Note that in Pirouz et al. actually two series of experiments are presented: one set for full-width flow in a half pipe and another set in a 1 m wide flume with self-formed channels of which channel widths are given. The latter series is utilized. These tests report the longitudinal slope at which the mud level stopped showing signs of sedimentation or erosion. Figure 3 shows calculated Reynolds numbers for the three criteria as a function of *W/h* and iterated wall shear stress.

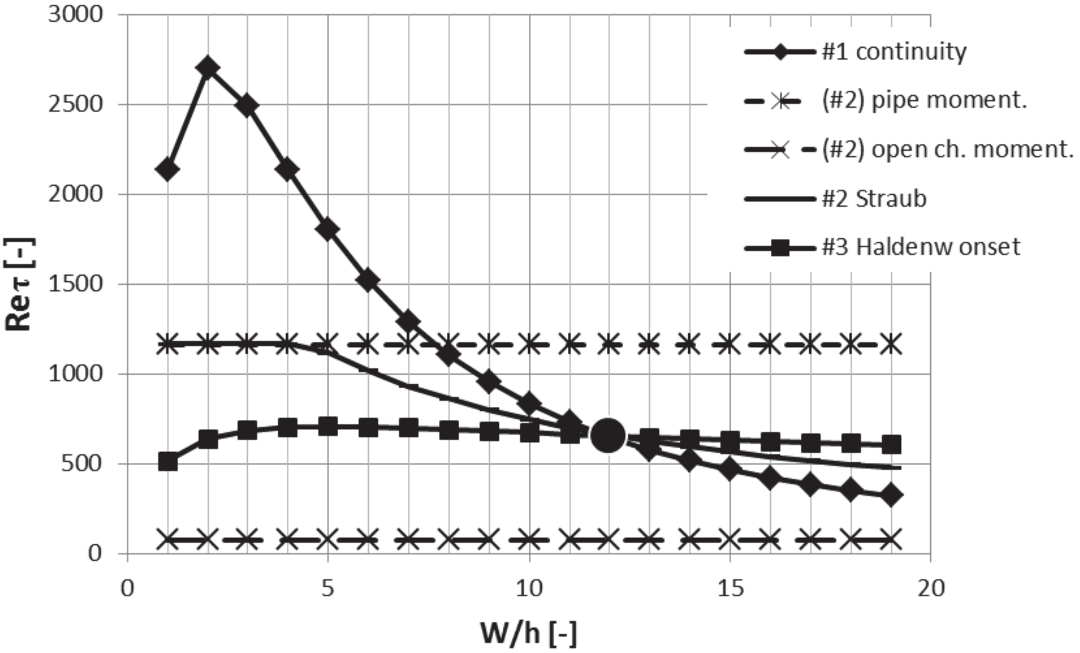

Fig.3 Criteria for channel conditions, i=0.046, Q=0.018 m^3/s, ρ=1625 kg/m^3, τ_y=12 Pa, K=1.53 Pas^n, n=0.45 at τ=23.5 Pa.

The curve representing continuity shows a maximum *Re* (or lowest *f*) at *W/h*=2. Two straight lines are shown for momentum balance theory according to pipe flow and wide open channel theory. In between, a weighing function is shown utilizing Straub's analogy for actual *W/h*. The last curve is the onset criterion as a function of *W/h*. The dot at the intersection of the curves indicates the condition satisfying all three criteria.

All calculation results are shown in Figure 4. The measured *W/h* ratio are determined from flow rate, measured velocity of the center plug and measured channel width. The majority of Pirouz et al. measurements are at a width-depth ratio (<15) which is closer to the pipe flow analogy than to the infinite wide channel geometry, Figure 1. Also calculation results for "classical" transition at *Re*=2000 are included, revealing differences at low *W/h*.

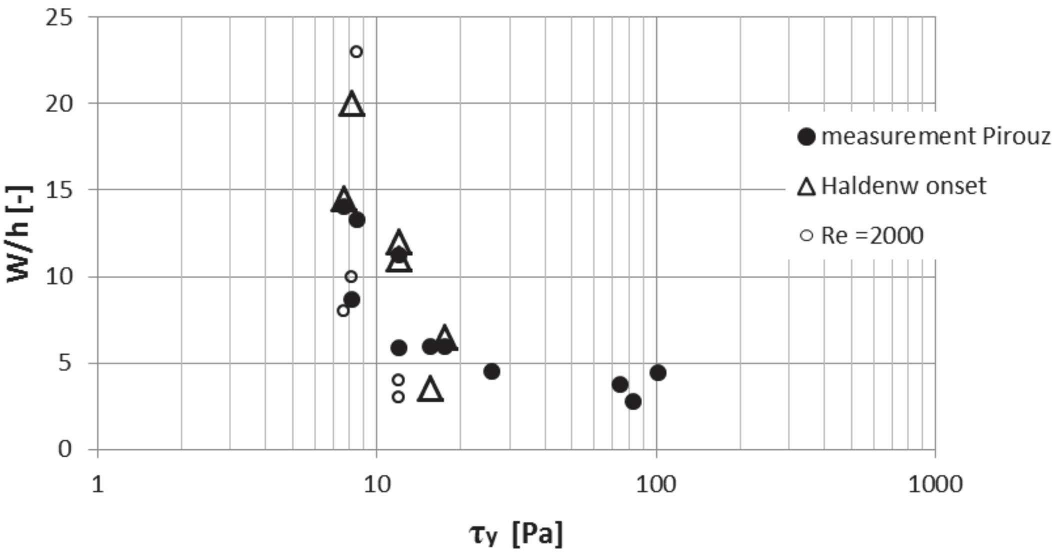

Fig.4 Predicted width/depth ratio of channels with proposed theory

It appears that onset-theory applies to moderate rheology conditions (τ_y < 20 Pa). For elevated rheologies this is not anymore the case and the flow is still laminar flowing in channels of minimal width-depth ratio, settling solids apparently being discharged.
Given the experience with current modeling it is hypothesized that smaller *W/h* ratio will result if the balance of erosion and sedimentation is reached deeper into the transition regime, see Fitton (2017). In that case the results might not be as sensitive to τ_y as currently predicted.

5. CONCLUSIONS

Channel width prediction is one of the essentials in deltaic deposition modeling in TSF's. Flow transition is important. The width-depth ratio of channels appears influencing the onset of transition as revealed by the Haldenwang data-set. The found influence of *W*/*h* on transition, suggest that non-Newtonian laminar flow is less stable than in infinite wide channels and pipes. Applying the correlation for onset of transition, satisfying continuity and satisfying the momentum balance, it is found that channel width of open channel flows can be predicted. With stronger rheological parameters onset conditions are not reached, but field data suggest that a minimum aspect-ratio of about *W*/*h*=4 is reached.

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