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A NEW CORRELATION FOR PARTICLE DIFFUSIVITY IN SLURRY FLOW THROUGH PIPELINE

Himanshu Pratap Singh, Shagun Pandey, D.R. Kaushal*

Civil Engineering Department, Indian Institute of Technology, New Delhi, India
**E-mail: kaushal@civil.iitd.ac.in*

A new correlation to determine the variation of dimensionless particle diffusivity with particle size and concentration across the pipe cross section is proposed considering the combination of modified Karabelas model for concentration profile and Wasp model for pressure drop. Durand's equation in Wasp model for determining the concentration at different points in the pipe cross-section is replaced by the modified Karabelas model in the present study. By doing so, the amount of particles moving in bed and vehicle portions of slurry are more accurately determined in the present study. A new correlation for dimensionless particle diffusivity is developed using concentration profile and pressure drop data obtained from the experiments, shows improvisation over the expressions proposed earlier.

KEYWORDS: pressure drop, slurry pipeline, concentration distribution, particle diffusivity

1. INTRODUCTION

Long distance transportation of slurries are generally used through pipes. It's been an endeavor for researchers designing and modelling of complex multisized slurry transporting pipelines. It involves various important flow parameters such as deposition velocity, concentration profiles and pressure drop. Therefore accurate prediction of these parameters using sufficient experimental data and design tools is always a priority.

The simple diffusion type model was first proposed by O'Brien (1933) and Rouse (1937) for the concentration profile prediction of solids in turbulent flow. Simple diffusion model was modified by Ismail (1952) by performing the correlation for mass transfer coefficient and stress/velocity gradient and that was deduced from Von Karman constant (K) and inversely proportional to the increasing solid concentration. Wasp (1963) found an agreement of his experimental data with the concentration profile while assuming K to be 0.35. Further Wasp et al. (1970) come up with a modified equation predicting concentration profile after analyzing their own data and Ismail (1952) data at y=0.08D and 0.5D, where D is the pipe diameter and y is the distance from pipe bottom. Shook and Daniel (1965) and Shook et al. (1968) found that the normal dispersive forces are not accounted in the equations proposed by O'Brien (1933) and Rouse (1937). An empirical model for the prediction of vertical composite concentration was developed by Karabelas (1977) on the basis of Hunt's (1954) formulation. Closed form expression by Karabelas (1977) also shown applicable by comparing the predicted values with experimental results. Equation proposed by Wasp et al. (1970) used by Seshadri et al. (1982) for the prediction of overall concentration profile along with individual size distribution and agreement up to large extent was found at the top of the pipe but deviations observed at the bottom of the pipe. Efforts are continuously being made by researchers to get more accurate concentration profile, such as Gillies et al. (1991) and Kaushal and Tomita (2002a,b). Most of the equations proposed earlier are empirical and produced using limited experimental data and for single sized or narrow size range of particles. Though in commercial slurries multisized particles are often encountered with heterogeneous particle distribution in turbulent flow. Considering all these points a model is proposed after certain modifications in the Karabelas (1977) and Wasp (1963).

2. DETAILED DESCRIPTION OF THE PROPOSED MODEL

Methodology used in the proposed model for the pressure drop calculation includes the application of modified Karabelas model for the estimation of pressure drop with Wasp model. Conventional approach to Wasp model suggests using following equation for obtaining the ratio of concentration at 0.08D from the top of pipeline and at the center line:

$$\log\frac{C}{C_A} = -1.8\frac{w_j}{\beta K u^*} \tag{1}$$

where C/C_A is the ratio of solids volumetric concentration at 0.08D from the top to that at pipe center, β is the particle dimensionless diffusivity and assumed as 1.0, K is Von Karman constant taken as 0.4, u* is the friction velocity and w_j is the settling velocity.

A computer program for the pressure drop calculation was developed and structured such that, concentration values at 0.92D from the pipe bottom and at the center line are obtained from modified Karabelas model (Kaushal and Tomita, 2002b) and further used in Wasp model to get the fraction of solids in vehicle and bed. In this combined model, shear velocity required to obtain the concentration profile is calculated simultaneously from the pressure drop calculated in each iteration. These iterations are performed to converge with ±1% error consideration and the pressure drop for the particular slurry flow are obtained.

Dimensionless particle diffusivity (β) proposed by Kaushal and Tomita (2002b) after analyzing multisized slurry flow experimental dataset and considering the effect of solid concentration and static settled concentration:

$$\beta = 1.0 + 0.12504\, e^{4.22054 C_{vf}/C_{vss}} \tag{2}$$

where C_{vss} is the static settled concentration and C_{vf} is the volumetric efflux concentration. Pressure drop due to bed is calculated using the Durand equation given as:

$$i_{jbed} = 82 i_w C_{vjbed}\left(\frac{gD(S-1)}{V_m^2\sqrt{C_D}}\right)^{1.5} \tag{3}$$

where i_w is the pressure drop due to carrier liquid, S is the specific gravity of solids, C_{vjbed} is the volumetric bed portion of j[th] particle size, C_D is the drag coefficient, D is the diameter of pipe, V_m is velocity of flow and g is acceleration due to gravity.
Darcy-Weisbach equation is used to calculate the vehicle pressure drop:

$$i_{vehicle} = \frac{f_m V_m^2}{2gD} \tag{4}$$

where f_m is the Darcy-Weisbach friction factor.

3. EXPERIMENTAL SETUP DETAILS

Experimental data used for analysis obtained from the experimental setup comprising pipeline, isokinetic sampling probe, pressure transducers, measuring tank, slurry mixing tank and slurry pump. Details of the setup and experimental procedure are explained elsewhere (Kaushal & Tomita, 2002a,b). Mild steel pipeline with internal diameter 105mm fabricated as a pipe loop, in which isokinetic sampling probe used to measure the composite and individual concentration profiles for six particle sizes ranging from 38 to 739μm using a traversing mechanism in vertical plane at nine levels for zinc tailings slurry. Flow of slurry at five efflux concentrations by volume varying from 4% to 26% at velocities 2, 2.75 and 3.5m/s each maintained.

4. COMPARISON OF PREDICTED RESULTS WITH THE EXPERIMENTAL DATA

The correlation when incorporated in the model, predicts the concentration profiles that are found to be in good agreement with the experimental data as shown in Fig. 2 and Fig. 3. Comparison of results obtained from modified Karabelas and proposed model is performed. β2002 represents the predicted results from the modified Karabelas model and β2017 represents the predicted results from the proposed model. Where $C_{vj}(y')$ is the individual volumetric concentration of different diameters plotted against the y'. y'=y/D, where y being the depth measured from the pipe bottom and D is the diameter of pipe. Variation of $C_{vj}(y')$ obtained from the proposed model are observed to be in the same fashion as the values obtained experimentally i.e. more skewed for the higher diameters and skewness reduces with decreasing diameters.

Total concentration of particles throughout the depth, calculated from the proposed model are in close vicinity of measured data, i.e. depicted in Fig. 3. Concentration predicted at almost all the locations except some location (i.e. at y=0.2D) near the bed showing improvement over the modified Karabelas model.

Pressure drop predicted by the proposed model increases with increase in velocity as shown in Fig. 1 and found to be in good agreement with the measured values of the pressure drop obtained from the experimental set up. Water data i.e. the pressure drop measured when only water is flowing through the pipeline is used to obtain the value of pipe roughness, which is further used to calculate friction factor for flow of mixture. Fig. 1

shows the pressure drop (i) measured experimentally, predicted from the proposed model and water data for eight values of flow velocities (V_m) ranging from 1.2 to 4m/s for total volumetric efflux concentrations (i.e. C_{vf}) 4%, 8.1%, 12.8%, 19.1% and 26% each.

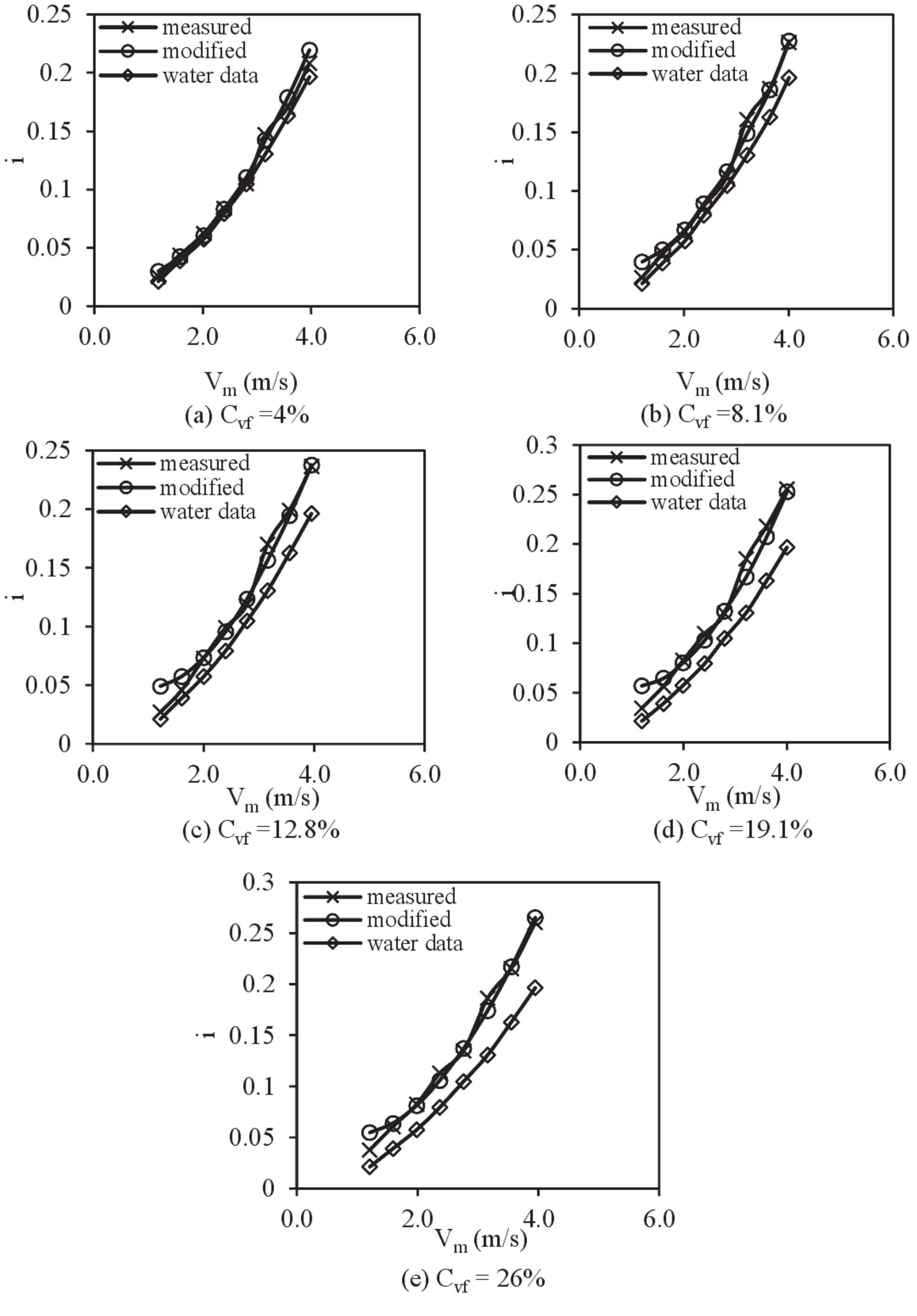

Fig.1 Comparison between measured and predicted pressure drops (i in meters of water column per meter of pipeline and V_m is the velocity of flow in m/s) for different volumetric efflux concentrations (C_{vf}) of zinc tailings slurry.

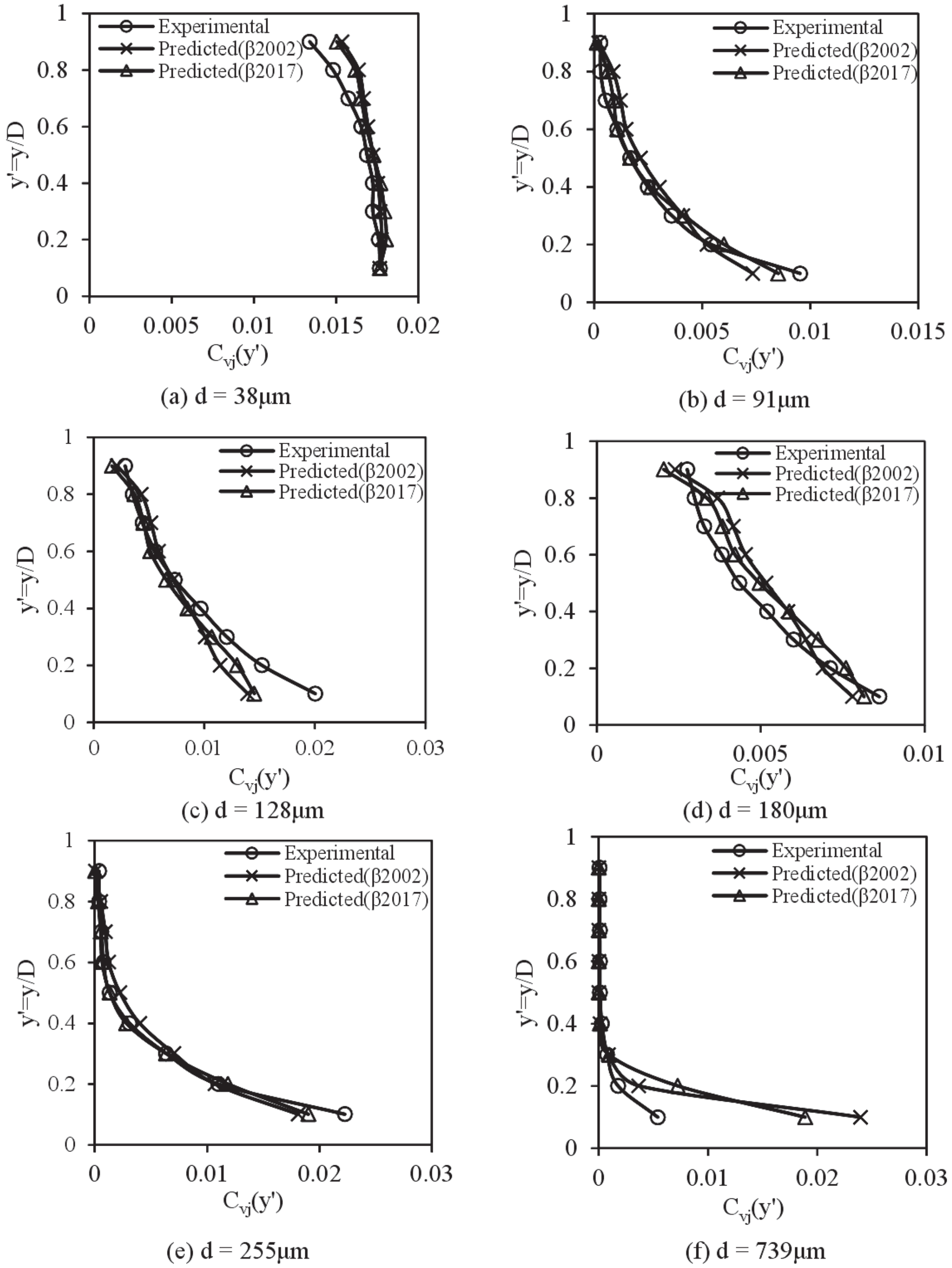

Fig. 2 Comparison between individual concentration profiles for all diameter particles (C_{vj}) obtained from the proposed model using previous (Kaushal and Tomita, 2002b) and modified β-correlation and the measured data for solids efflux concentration 4% by volume at the flow velocity 2m/s.

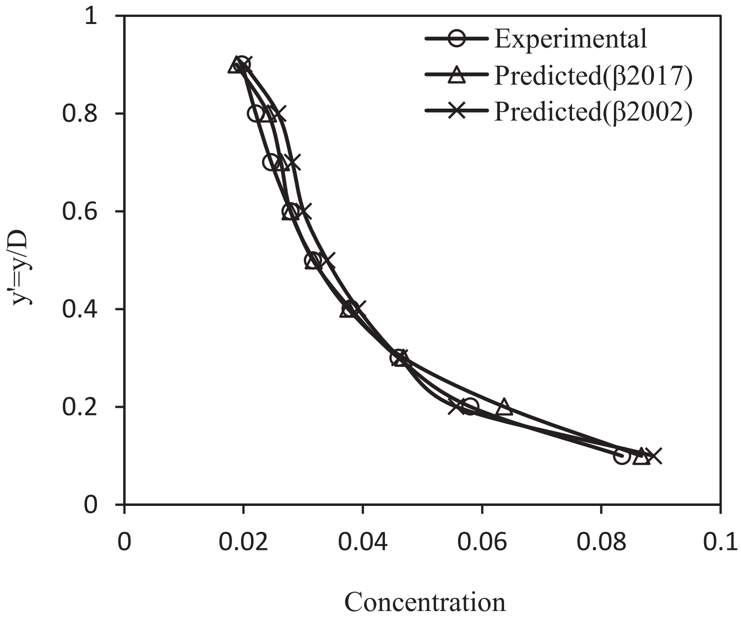

Fig. 3 Total concentration at different points throughout the cross section for previous (Kaushal and Tomita, 2002b) and proposed correlation compared with experimental data for a flow velocity of 2m/s and total efflux concentration 4%.

5. OBTAINING THE OPTIMIZED VALUE OF PARTICLE DIFFUSIVITY

Mathematical model prepared using MATLAB tool is further coded in order to get the optimum values of particle diffusivity. Optimum values of particle diffusivity are obtained at nine location points from top to bottom i.e. from 0.1D to 0.9D for the different particle sizes used in the experiments, where D is diameter of pipe. It is achieved by calculating the value of particle diffusivity that exactly corresponds to the measured solids volumetric concentration of individual particle size. These optimum values obtained can now be used to generate a new particle diffusivity correlation.

For generating correlation, variation of optimum particle diffusivity (i.e. β) values are observed with depth and diameter. Since the particle diffusivity correlation proposed by Kaushal and Tomita (2002b) accounts only the variation of efflux concentration and static settled concentration, now we need to get a factor depending on depth (y) and particle diameter (d) to be used in the same.

$$\beta_{2017} = \beta_{2002} \times \alpha \qquad (5)$$

β_{2002} is the particle diffusivity correlation proposed by Kaushal and Tomita (2002b) as mentioned in equation (2) and β_{2017} is the required correlation for proposed model. Factor α is obtained using least square curve fitting method as shown in Fig. 4, where d is the diameter of particle in microns (μm) and equation (6) is the correlation obtained for the variation of dimensionless particle diffusivity coefficient (β) with particle diameter at two locations (i.e. y=0.1D and y=0.9D).

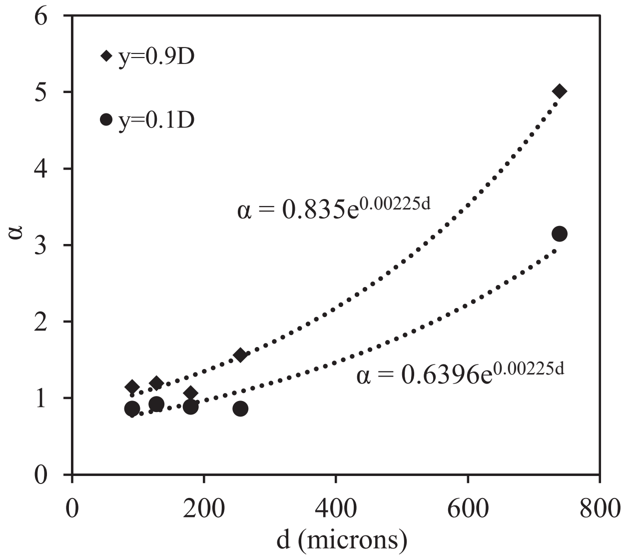

Fig. 4 α vs particle diameter (d) curve.

Constants obtained from the exponential functions shown in Fig. 4 are now further used to correlate with y' (i.e. y/D) and obtain the final expression as shown below:

$$\beta_{2017} = (1 + 0.125e^{4.22054 C_{vf}/C_{vss}})[(0.2443(y/D) + 0.6156)(e^{0.00225d})] \quad (6)$$

In the previous methods, the optimization of β performed regarding the concentration profile i.e. comparing the predicted concentration with experimental observations, but in the proposed model pressure drop is also been considered throughout the methodology, which also results in a whole new trend of variation of β. Here, β is interestingly found to be varied with vertical distance from the pipe bottom and diameter of particle as well. By incorporating the proposed correlation, pressure drops are predicted more accurately, on the other hand the values of individual particle concentration (i.e. C_{vj}) and total concentration are also modified i.e. closer to the experimental data.

6. CONCLUSION

1. This model is more convenient to use, since both concentration profile and pressure drop can be calculated simultaneously and the experiments need not to be done, as the model is purely analytical.
2. The concentration profile and pressure drop values obtained from the model shows satisfactory agreement with the experimental data referred from Kaushal and Tomita (2002a).

3. The new improvised Wasp model, which is the replacement of the empirical formula with the modified Karabelas's concentration ratio improve the accuracy of the pressure drop values in the pipe flow.
4. Variation of particle diffusion coefficient (β) along the vertical cross section of pipe and the particle size is also considered while obtaining the correlation, that were not considered in the earlier research works.

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