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PREDICTING PARTICLE SETTLING RATE IN A SHEARED MINING SLURRY

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Many tailings slurries are comprised of fine particles suspended in a carrier fluid. For high enough concentrations, the fines and water combine to form a non-Newtonian slurry. In a number of important applications, these slurries also contain larger, denser particles that are able to settle under shear. This results in inhomogeneity in the flow and can potentially significantly increase the pressure gradient required to drive it. A key unanswered question is how the settling rate of large particles is related to the rheology of the fluid and the local shear in the particle vicinity. In this paper, a numerical investigation of large particle settling in un-sheared and sheared mining slurries is presented. For power-law rheology, the ratio of sheared to un-sheared particle settling rate increases with increasing imposed shear and with increasing shear thinning. We propose a criterion for estimating the settling velocity under shear that can be used to provide estimates of particle settling during transport and the likely distance before complete stratification in laminar flow.

KEY WORDS: Particle settling; solids transport; sheared settling; viscoplastic slurry

1. INTRODUCTION

In high concentration, fine particle suspensions, a knowledge of the terminal velocity of large dense particles in the usually non-Newtonian carrier is desirable. Examples include transport of mine tailings, removal of swarf in oil and gas well drilling, concrete pumping and transport of food materials with inclusions. These fluids exhibit non-Newtonian behaviour and their rheology is often well-modelled using simplified rheology models. Because the coarse particle density is usually greater than the density of the carrier media, the solids can potentially settle, resulting in an accumulation in pipelines or machines. Because the carrier rheology is shear dependent, settling velocity will depend on the flow state, thus an understanding of how shear affects settling is needed to predict settling rates, times and distances in pipelines or process equipment.

Previous studies have reported settling velocity and drag coefficient for spherical particles settling in non-Newtonian fluids. The experimental and numerical works of Graham and Jones (1994), Atapattu et al. (1995) and Dhole et al. (2006) provided a prediction of settling velocities in unsheared power law fluids. They compare well with each other with a maximum deviation of around 11% for higher particle settling Reynolds number ($O(100)$). With an imposed shear, Ovarlez et al. (2012), Gheissary and van den Brule (1996) and Talmon et al. (2014) considered different ways of imposing an additional shear to settling particles and all found that it aided the ability of particles to settle. Little published research is available on large particle settling velocity predictions in fluids with the properties of mining slurries and to our knowledge no work has been done on predictions under plane sheared conditions.

In this study, we present a computational model that allows simulation of coarse particle settling in sheared fluids. In practice the coarse particle fraction in a slurry is unlikely to be so dilute that particles can be considered in isolation. However before finite volume fractions can be considered, the effect of shear on an isolated particle must be categorised and that is the aim of the present study. We predict the settling velocity and drag coefficient of a one millimetre particle settling in power law fluids that are representative of mining slurries, and consider density differences between in the range 100–4000 kg m^{-3}. With this model, we are able to predict the settling behaviour at different applied shear rates, providing a basis for more complex situations.

2. NUMERICAL METHOD

2.1. GOVERNING EQUATIONS AND RHEOLOGY MODEL

The equations of motion for an incompressible laminar flow of a power law fluid are:

$$\frac{\partial u_i}{\partial x_i}=0; \qquad \rho\left(\frac{\partial u_i}{\partial t}+\frac{\partial u_i u_j}{\partial x_j}\right)=-\frac{\partial P}{\partial x_i}+\frac{\partial \tau_{ij}}{\partial x_i}; \qquad \tau_{ij}=\eta\left(\dot{\gamma}\right)S_{ij}; \qquad \dot{\gamma}=\sqrt{\tfrac{1}{2}S_{ij}S_{ij}} \tag{1}$$

where u_i is the component of the velocity in the x_i–direction, ρ is the fluid density, η is the effective viscosity, P is the pressure and τ_{ij} is the stress tensor. The rate of deformation tensor is $S_{ij} = \nabla u + \nabla u^T$. The power law rheology model is then written

$$\eta = k\,\dot{\gamma}^{n-1} \tag{2}$$

where k is the consistency and n the flow index. Choosing a density scale given by the fluid density, a length scale equal to the particle diameter d, a velocity scale given by the settling velocity V, and a viscosity scale given by the viscosity at a shear rate of V/d, the dimensionless form of the momentum equation leads to the particle settling Reynolds number, Re:

$$\mathrm{Re}=\frac{\rho d^n V^{2-n}}{k} \tag{3}$$

The equations of motion (Eqn. 1) are solved using the finite-volume based solver *simpleFoam*, a branch of OpenFOAM 3.1.0.

2.2. COMPUTATIONAL DOMAIN

We approximate the idealised problem of a particle settling in an infinite medium by using a single sphere at the centre of a cubic box with side length D. To reduce computational complexity we perform the simulation in a frame of reference attached to the particle, thus the walls of the box move upwards with respect to the stationary particle. Figure 1 shows a schematic representation of the computational set-up. The left and right (x) walls of the box are defined as no-slip boundaries that move vertically ($y-$direction) with a speed equal to the settling velocity $V-$ this will drive a flow around the sphere that is equivalent to the flow due to particle settling. However V is the settling velocity we seek in the first place, hence an algorithm to find the correct velocity is required and is discussed below.

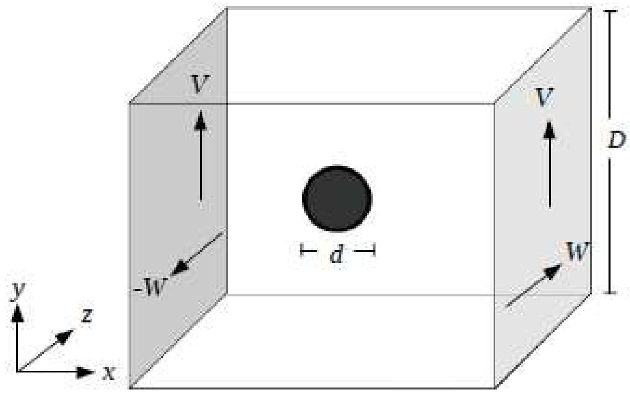

Fig. 11 Schematic of the computational domain

Periodic boundary conditions are used on the two other pairs of walls (top and bottom (y) and front and back (z)) of the box. This approximation is equivalent to simulating an infinite 2D array of particles settling between two infinite moving plates and has the potential to introduce errors. The effect of this approximation is also discussed below. As described, the model is appropriate for investigating unsheared particle settling. For an applied shear, the left and right $x-$walls of the domain are also moved in the opposite direction with velocity W in the z-direction in addition to V in y-direction. Therefore, the boundary conditions for the left and right walls are (0 V -W) and (0 V W) respectively, giving an applied shear of $2W/D$.

Independence of the solution for both the grid resolution and computational domain size was considered. To study the effect of resolution, a fixed box size (10D big) was used and key geometric parameters were modified: the number of sphere surface elements (Ns), the thickness of the first shell of elements on the sphere surface (Ns) and the expansion factor for the element thickness moving away from the surface ($1 + e$). Predictions of the drag coefficient C_D were compared to determine when the results converged. Values of $Ns \geq 10000$, $\delta s =$ one tenth of the length scale of the surface mesh and $e = 0.3$ were subsequently used in the simulations. Predictions of the drag coefficient C_D for different box sizes (at the converged resolution mentioned above) were also compared. From this it was determined that the ratio of domain size (D) to particle size (d) needed to be ≥ 15 to provide converged results. This result is in agreement with the results presented in

Atapattu et al. (1995) for yield stress fluids. Approximately 300K grid cells were used in the simulations.

2.3. DETERMINING SETTLING VELOCITY AND ROTATION RATE

For steady settling, there is zero net force on the particle and the drag experienced by the particle will be balanced by $(\pi/6)\Delta\rho\, d^3 g$. For a given particle size, density difference and fluid rheology, a unique velocity, V, must be found for which the net force is zero. This velocity cannot be directly estimated and is found here using the secant root finding method with an initial guess for V provided by the Stokes settling velocity in an equivalent Newtonian fluid. For the case of an imposed shear, the particle will also rotate about the $y-$ axis. For a Newtonian fluid in the Stokes regime this rotation rate is given by half the imposed shear, i.e. $\omega = W/D$. For power-law fluids and/or non-zero Reynolds number, this rotation is unknown. In the simulation, a rotational velocity about the y–axis ω must be applied as boundary condition on the sphere so that the net torque on the sphere is zero. The rotational velocity that provides zero net torque is again found by using the secant root finding method with an initial guesses of $\omega = W/D$.

The rheology parameter space considered in this study was based on the power law rheological data for a range of mining slurries obtained from Sofra and Boger (2002); Turian et al. (2002); Bakker et al. (2009). The range of consistency (Pa s^n) is $0.1 < k < 2$ and for flow index is $0.3 < n < 0.7$.

3. PARTICLE SETTLING PREDICTIONS

Settling of a one millimetre particle with particle-liquid density differences in the range 100–4000 kg/m^3 was considered in this study. We present the results in two parts. In the first, we consider settling without imposed shear and compare our results to the experimental results of Graham and Jones (1994). In the second part, we consider how imposed planar shear modifies the settling velocity and drag coefficient.

3.1. PARTICLE SETTLING IN AN UNSHEARED POWER-LAW FLUID

For the parameters considered here, the particle settling Reynolds numbers ranged from $10^{-6} \leq Re \leq 225$. An example of a typical flow field is shown in Figure 2a and 2c. Figure 2a shows the variation of the $y-$ component of velocity. Its magnitude is zero around the sphere's surface (due to the no-slip boundary condition), and its distribution increases uniformly slowly away from the sphere. As can be seen in Figure 2c, the viscosity is unsurprisingly lowest around the sphere surface (except at the rear stagnation point) and the high-viscosity exterior fluid pinches in at the equator, indicating low shear at the particle sides. The comparison to sheared particle settling in Figure 2b and 2d will be discussed in the next section.

As expected, particle settling velocity increases for increasing density difference as shown in Figure 3a. Unsurprisingly, the effect of fluid consistency k on V_U is approximately linear over the range of Re (10^{-6} to 200) covered in this study, with higher consistency resulting in lower velocity. The effect of flow index is to increase settling velocity as n decreases.

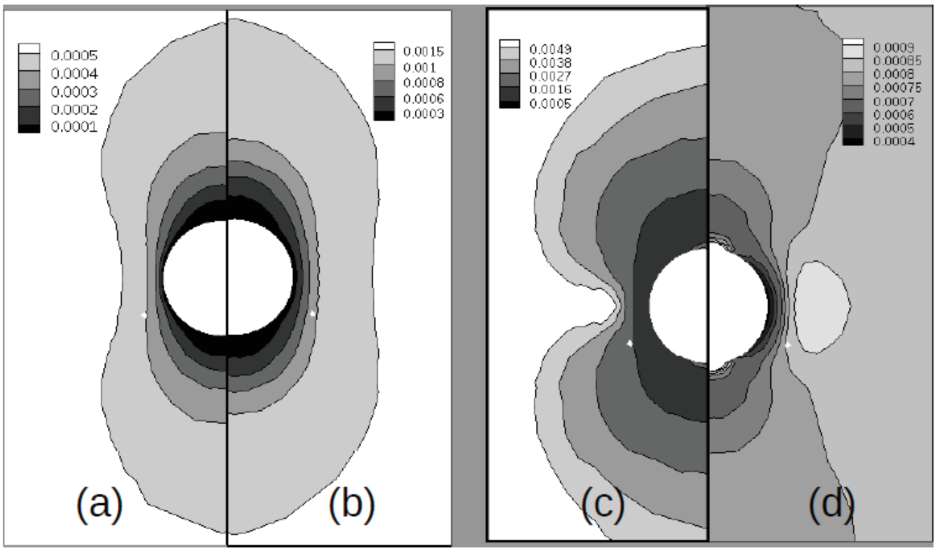

Fig. 2 Contours of (a) unsheared velocity y component (b) sheared velocity y component (δ=10) (c) unsheared viscosity distribution (d) sheared viscosity distribution (δ = 10) (for $k = 2Pa\ s^n$, n = 0.5 and $\Delta\rho$ = 4000)

Graham and Jones (1994), Atapattu et al. (1995) and Dhole et al. (2006) all presented relationships between C_D and Re for power law fluids at a finite Re. The results presented in Figure 3a were appropriately non-dimensionalised and plotted in Figure 3b to compare. The results here show good agreement with the previously published correlations. This agreement provides clear evidence of the reliability of the computational model, and confidence that subsequent results for settling under imposed shear will be similarly reliable.

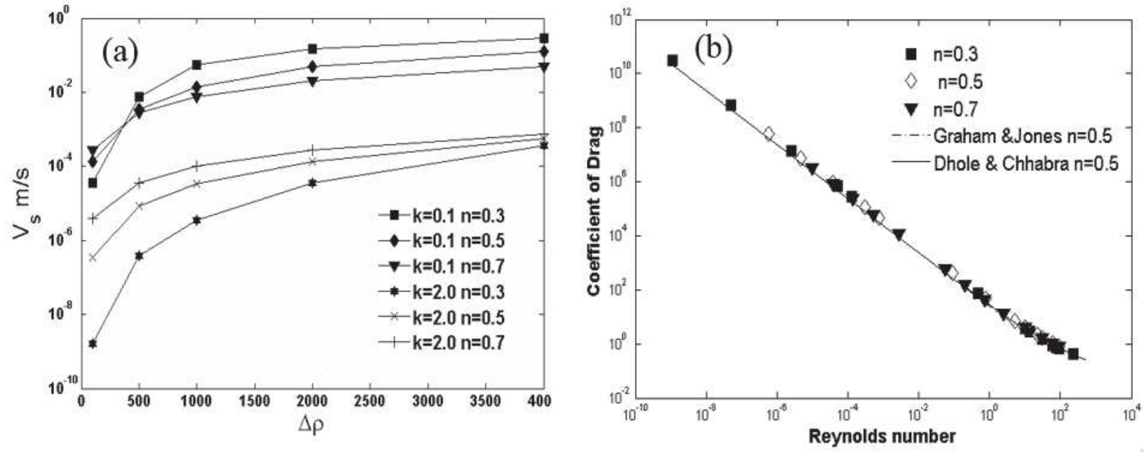

Fig. 3. (a) Settling velocity as a function of density difference (b)Variation of drag coefficient with Re for 0.3 < n < 0.7. Comparison to Graham and Jones (1994).

3.2. PARTICLE SETTLING UNDER IMPOSED SHEAR

With an imposed shear, the local shear rate in the entire flow, including the particle vicinity, is affected. Since viscosity is power law function of shear rate , increasing shear rate causes viscosity to decrease around the particle. It seems likely that this will result in a faster settling velocity as a result of decreased effective viscosity. The imposed shear from the moving walls here is set as a multiple of the shear rate scale determined from unsheared settling. Thus we define δ as the ratio of the shearing plane shear rate and the unsheared settling shear rate which is given by Eqn. 4:

$$\delta = \frac{2W/D}{V_U/d} \tag{4}$$

where W is the shearing plane velocity, D is the domain size, d is the particle size and V_U is the unsheared settling velocity.

Results for a one millimetre particle settling in sheared power law fluids with n = 0.3 and 0.7 for $100 < \Delta\rho < 4000$ and $0.1 < \delta < 10$ are shown in Figure 4. As can be seen, for a given rheology, as the imposed shear rate ratio δ increases, the particle settling velocity also increases. As expected, the impact of imposed shear is to decrease the viscosity across the entire computational domain, but the local effect in the vicinity of the particle enables the particle to settle faster. With this in mind, we proceed to quantify the change in settling velocity as a function of the rheology and δ. For one rheology, a comparison between the sheared flow field (δ = 10) and unsheared flow field is shown in Figure 2. Clearly seen is a significant increase in velocity and decrease in viscosity compared to that in the unsheared case.

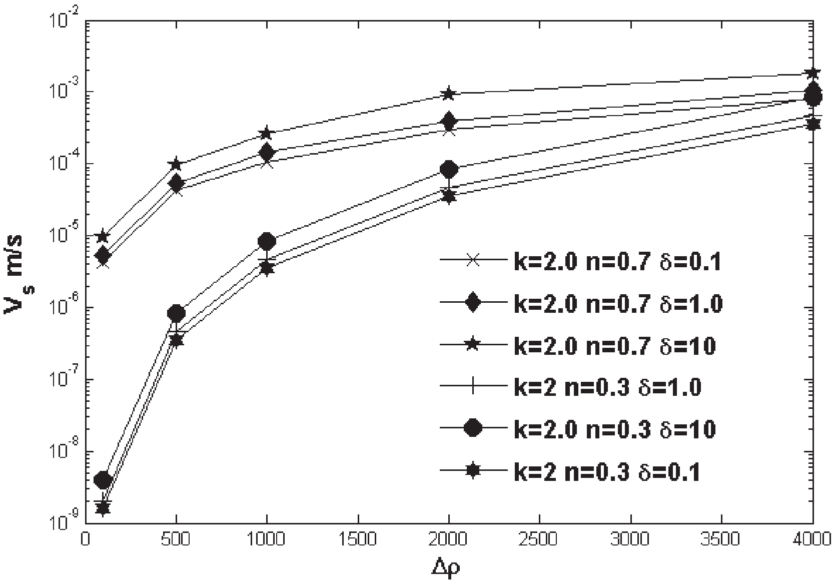

Fig. 4. Settling velocity as a function of Δρ for different δ.

3.3. RATIO OF SHEARED TO UNSHEARED SETTLING VELOCITY

We present the results of sheared settling in terms of the ratio of velocity in the case of imposed shear to that in the unsheared case. Results of V_{Sh}/V_U for a given value of k (2.0 Pa s^n) are shown in Figure 5a as a function of imposed shear rate ratio δ. Important to note is that each of the curves in Figure 5a is indistinguishable for every density ratio considered, i.e. the results in Figure 4 for a fixed n collapse for all $\Delta\rho$. This then means that the results appears to be independent of Re, at least over the range of Re of the simulations(10^{-6} to 200). The second obvious result is that just like the results in Figure 4 for V_{Sh} the ratio V_{Sh}/V_U increases with decreasing n – more shear thinning results in faster relative settling in a sheared fluid. Although not obvious from Figure 5a, our results also predict that the settling velocity of a Newtonian fluid is affected by applied shear, although this is quite a weak effect being $V_{Sh}/V_U = (1 + 0.003\,\delta)$.

The effect of consistency k (0.1 or 2) on the ratio V_{Sh}/V_U is shown in Figure 5b for a single flow index $n = 0.5$. Again these results cover the full range of density differences. Clearly seen is that k has no direct influence on the ratio of sheared to unsheared settling velocity which is a consequence of the viscosity being a linear function of k. It suggests again the ratio V_{Sh}/V_U is not a strong function of Re.

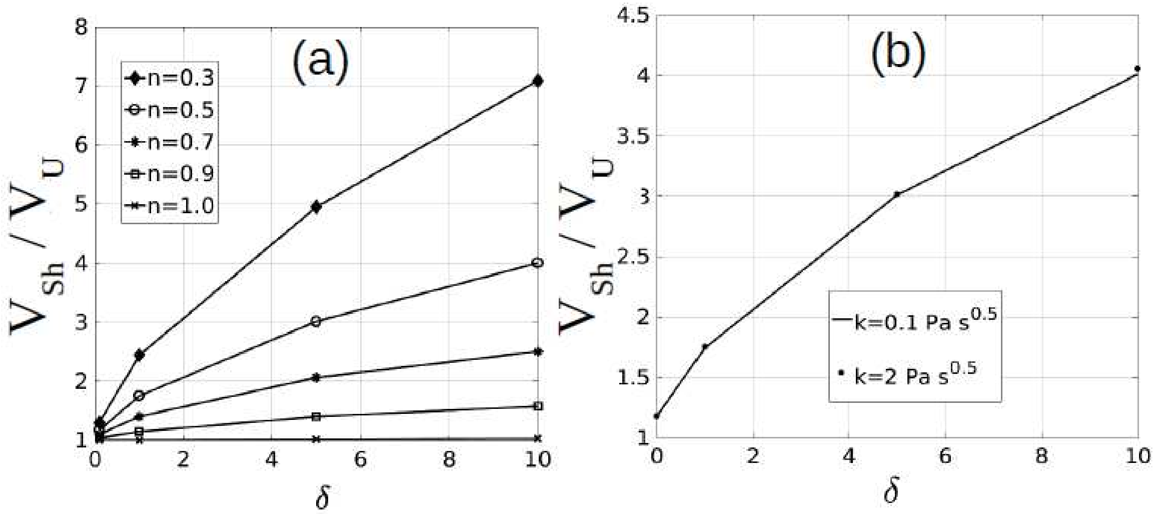

Fig. 12 V_{Sh}/V_U as a function of δ for (a) flow index 0.3 < n < 1.0 at constant k = 2.0 and (b) for k=0.1 and 2 Pas^n at constant n = 0.5

From the results for imposed shear, we observe that the velocity ratio is primarily a function of imposed shear and flow index. Based on this understanding we collapse the data to determine the following unified correlation:

$$\frac{V_{Sh}}{V_U} = \left(1+0.003\delta\right)\left(1+1.56\delta\right)^{1-n} \tag{5}$$

Equation 5 incorporates the effect of shear due to both settling and the imposed shear through δ and was obtained using non-linear least square regression analysis. The first term arises from a need to recover the Newtonian result as n approaches unity. The functional form of the second term is based on the ratio of viscosity scale η_{Sh} in the case of imposed shear to that in the unsheared case η_U (see Equation 6 for the definition of these scales).

$$\eta_{Sh} = k\left(\sqrt{\left(\frac{V_{Sh}}{d}\right)^2 + \left(\frac{\delta V_U}{d}\right)^2}\right)^{n-1} \qquad \eta_U = k\left(\frac{V_U}{d}\right)^{n-1} \tag{6}$$

This ratio of viscosity scales can be shown to take the form $(1 + a\,\delta)^{1-n}$ and is thus used to determine the correlation.

4. CONCLUSIONS

In the present study, particle settling in unsheared and sheared power-law fluids has been investigated. Numerical predictions for unsheared settling were found to be in good agreement with past studies. A detailed examination of computational results showed how shear rate increases and viscosity decreases close to the particle surface. These predictions are useful not only in estimating the settling velocity but also provide useful insights on details of the flow. In the case of an imposed shear, we show that the particle settling velocity increases with increasing shear rate as expected and we propose a criterion for estimating the settling velocity under plane shear (Equation 5). This result has implications for settling time and importantly the settling distance required for a homogeneous coarse particle suspension to become stratified. This study will be further extended to consider at the effect of shear rates for various particle size, shear thinning and yield stress fluids.

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