

THE INFLUENCE OF LOCAL HINDERED SETTLING ON THE CONCENTRATION DISTRIBUTION IN SLURRY TRANSPORT

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For determining the concentration distribution in slurry transport in pipes, often the solution of the 2D advection diffusion equation is used. The solution contains the terminal settling velocity, which is often replaced by the global hindered settling velocity, based on the cross sectional averaged spatial volumetric concentration. The fact that a concentration distribution is determined implies that the local concentration depends on the vertical coordinate in the pipe and is not equal to the cross sectional averaged concentration. With CFD this influence can be determined, however it's interesting to see if there is also an analytical solution to this. The paper describes this analytical model based on local concentration. The method is iterative and requires about 4-12 iterations to find a stable solution. The hindered settling equation of Richardson and Zaki is modified to find a correct hindered settling velocity for high concentrations. In fact, if the spatial concentration approaches 50% one cannot speak of hindered settling anymore, but its closer to consolidation. The new equation behaves according to Richardson & Zaki for small concentrations and to consolidation for large concentrations. The model is validated with measured concentration profiles from literature, which will be shown.

KEY WORDS: slurry transport, concentration distribution, hindered settling.

NOTATION

vehicle	Pseudo liquid formed by carrier liquid and suspended particles	-
C	Integration constant	-
$C_{vs}(r)$	Volumetric concentration as a function of vertical coordinate in pipe	-
$C_{vs,v}$	Volumetric concentration in vehicle	-
C_{vb}	Volumetric concentration bed	-
C_{vB}	Volumetric concentration bottom of channel or pipe	-
C_{vr}	Relative volumetric concentration	-
d	Particle diameter	m
D_p	Pipe diameter	m
g	Gravitational constant (9.81 m/s ²)	m/s²
r	Vertical coordinate in pipe	m

$\tilde{r}$	Dimensionless vertical coordinate in pipe	-
R	Radius of pipe	m
R_{sd}	Relative submerged density (about 1.65 for sand)	-
u^*	Friction velocity	m/s
u^*,ldv	Friction velocity at LDV	m/s
v_{ls}	Line speed	m/s
$v_{ls,ldv}$	Limit Deposit Velocity	m/s
v_t	Terminal settling velocity	m/s
v_{tv}	Terminal settling velocity in vehicle	m/s
$v_{tv,ldv}$	Terminal settling velocity in vehicle at LDV	m/s
v_{th}	Terminal settling velocity hindered	m/s
$v_{th,ldv}$	Terminal settling velocity hindered at LDV	m/s
v_{thv}	Terminal settling velocity hindered in vehicle	m/s
$v_{thv,ldv}$	Terminal settling velocity hindered in vehicle at LDV	m/s
α_{sm}	Correction factor	-
α_{c1}	Correction factor	-
α_{c2}	Correction factor	-
β	Richardson & Zaki power hindered settling	-
β_{sm}	Relation sediment mass diffusivity to eddy momentum diffusivity.	-
$\beta_{sm,ldv}$	Relation sediment mass diffusivity to eddy momentum diffusivity at LDV	-
ε_m	Eddy momentum diffusivity	m/s
ε_s	Sediment diffusivity	m/s
$\bar{\varepsilon}_s$	Average sediment diffusivity	m/s
λ_1	Darcy Weisbach friction factor	-
$\lambda_{1,ldv}$	Darcy Weisbach friction factor at LDV	-
κ	von Karman constant	-
φ	Pipe angle	°

1. INTRODUCTION

In literature, the concentration distribution is often determined with the solution of the advection diffusion differential equation for 2D channel flow. The advection diffusion equation is derived and solved for low concentrations and for high concentrations including the effect of upwards liquid flow. Wasp et al. (1977) and Doron et al. (1987) use the solution for low concentrations, while Karabelas (1977) and Kaushal & Tomita (2002B) use the Hunt (1954) approach with upwards liquid flow. Hindered settling is not yet included in the basic solutions, but added by replacing the terminal settling velocity with the hindered terminal settling velocity. For the diffusivity and the relation between the sediment diffusivity and the turbulent eddy momentum diffusivity different approaches are possible.

If we assume the diffusivity as a constant, the advection diffusion equation can be solved. Giving the differential equation in the equilibrium situation with hindered settling, but without the upwards liquid velocity, since this is assumed to be part of the hindered settling (with r the vertical distance from the bottom of the pipe):

$$C_{vs}(r) \cdot v_{th} + \varepsilon_s \cdot \frac{dC_{vs}(r)}{dr} = C_{vs}(r) \cdot v_{th} + \beta_{sm} \cdot \varepsilon_m \cdot \frac{dC_{vs}(r)}{dr} = 0 \quad (3)$$

The coordinate r ranges from 0 to D_p , the pipe diameter. Now the variables must be separated according to:

$$\frac{dC_{vs}(r)}{C_{vs}(r)} = -\frac{v_{th}}{\beta_{sm} \cdot \varepsilon_m} \cdot dr \quad \Rightarrow \quad \ln(C_{vs}(r)) = -\frac{v_{th}}{\beta_{sm} \cdot \varepsilon_m} \cdot r + C \quad (4)$$

With $C_{vs}(0)=C_{vB}$, the bottom concentration, the integration constant can be determined, giving:

$$C_{vs}(r) = C_{vB} \cdot e^{-\frac{v_{th}}{\beta_{sm} \cdot \varepsilon_m} \cdot r} \quad (5)$$

This basic solution is still equal to the solution for open channel flow. Although this is just an indicative equation for open channel flow, Doron et al. (1987) and Doron & Barnea (1993) used it in their 2 and 3 layer models. The difference between pipe flow and open channel flow is in the determination of the diffusivity. Assuming the Law of the Wall, one can also determine the average diffusivity by integration (Lane & Kalinske (1941)):

$$\varepsilon_s = \beta_{sm} \cdot \kappa \cdot u_* \cdot r \cdot \left(\frac{R-r}{R} \right) = \beta_{sm} \cdot \kappa \cdot u_* \cdot R \cdot \frac{r}{R} \cdot \left(1 - \frac{r}{R} \right) = \beta_{sm} \cdot \kappa \cdot u_* \cdot R \cdot \tilde{r} \cdot (1 - \tilde{r}) \quad (6)$$

Integration over the cross section of the pipe gives:

$$\bar{\varepsilon}_s = \frac{\beta_{sm} \cdot \kappa \cdot u_* \cdot D_p}{12} \quad (7)$$

This gives for the concentration as a function of the vertical distance from the pipe bottom:

$$C_{vs}(r) = C_{vB} \cdot e^{-12 \cdot \frac{v_{th}}{\beta_{sm} \cdot \kappa \cdot u_*} \cdot \frac{r}{D_p}} \quad (8)$$

Based on integration, assuming open channel flow and settling in the carrier liquid, the bottom concentration C_{vB} can be found, at the Limit Deposit Velocity (LDV) $C_{vB}=C_{vb}$:

$$C_{vB} = C_{vs} \cdot \left(\frac{12 \cdot v_{tv}}{\beta_{sm} \cdot \kappa \cdot u_*} \right) \Bigg/ \left(1 - e^{-12 \cdot \frac{v_{tv}}{\beta_{sm} \cdot \kappa \cdot u_*}} \right) \quad (9)$$

2. THE DIFFUSIVITY BASED ON THE LDV

To have a consistent model, the bottom concentration should be equal to the bed concentration at the LDV, since this is the definition of the LDV. Now most concentration profile equations are not related to the LDV, but make use of a modified diffusivity to match experiments. Here an attempt is made to make the concentration profile match the LDV. At the Limit Deposit Velocity, the bottom concentration C_{vB} equals the bed concentration C_{vb} giving:

$$C_{vb} = C_{vs} \cdot \left(\frac{12 \cdot v_{tv,ldv}}{\beta_{sm,ldv} \cdot \kappa \cdot u_{*,ldv}} \right) \left/ \left(1 - e^{-12 \cdot \frac{v_{tv,ldv}}{\beta_{sm} \cdot \kappa \cdot u_{*,ldv}}} \right) \right. \quad (10)$$

Neglecting the denominator at low concentrations, since it's close to unity (say the denominator equals a factor α_{sm}), the diffusivity can be derived.

$$\beta_{sm,ldv} = 12 \cdot \frac{C_{vs}}{C_{vb}} \cdot \frac{v_{tv,ldv}}{\alpha_{sm} \cdot \kappa \cdot u_{*,ldv}} = 12 \cdot C_{vr} \cdot \frac{v_{tv,ldv}}{\alpha_{sm} \cdot \kappa \cdot u_{*,ldv}} \quad (11)$$

This gives for the concentration distribution in the pipe:

$$C_{vs}(r) = C_{vB} \cdot e^{-12 \cdot \frac{v_{tv}}{\left(12 \cdot C_{vr} \cdot \frac{v_{tv,ldv}}{\alpha_{sm} \cdot \kappa \cdot u_{*,ldv}} \right) \cdot \kappa \cdot u_{*}} \cdot \frac{r}{D_p}} = C_{vB} \cdot e^{-\frac{\alpha_{sm} \cdot u_{*,ldv}}{C_{vr} \cdot u_{*}} \cdot \frac{v_{tv}}{v_{tv,ldv}} \cdot \frac{r}{D_p}} \quad (12)$$

The bottom concentration is, using the fact that the denominator equals the factor α_{sm} :

$$C_{vB} = C_{vb} \cdot \frac{u_{*,ldv}}{u_{*}} \cdot \frac{v_{tv}}{v_{tv,ldv}} \quad (13)$$

The correction factor has to be determined at the LDV, giving an implicit equation with only the relative volumetric concentration as the parameter:

$$\alpha_{sm} = \left(1 - e^{-\frac{\alpha_{sm}}{C_{vr}}} \right) = 0.9847 + 0.304 \cdot C_{vr} - 1.196 \cdot C_{vr}^2 - 0.5564 \cdot C_{vr}^3 + 0.47 \cdot C_{vr}^4 \quad (14)$$

At low relative concentrations, $C_{vr} < 0.3$, this factor is about 1. Based on the diffusivity derived, the portion of the solids in the vehicle (suspension) according to the Wasp criterion can be determined by:

$$\frac{C_{vs,v}}{C_{vs}} = e^{-\left(0.92-0.5\right) \cdot \frac{\alpha_{sm}}{C_{vr}} \cdot \frac{u_{*,ldv}}{u_{*}} \cdot \frac{v_{tv}}{v_{tv,ldv}}} \quad (15)$$

3. HINDERED SETTLING NUMERICAL IMPLEMENTATION

The concentration profiles found match well except for the influence of local hindered settling, which will decrease the concentration at the top of the pipe and increase the concentration at the bottom of the pipe.

One of the main issues is that the Richardson & Zaki (1954) hindered settling equation is based on the spatial volumetric concentration C_{vs} and not on the relative spatial volumetric concentration $C_{vr}=C_{vs}/C_{vb}$. Their equation yields:

$$\frac{v_{th}}{v_t} = (1 - C_{vs})^\beta \quad (16)$$

So even when the spatial volumetric concentration reaches a concentration where a bed with maximum porosity occurs, for sand at about $C_{vs}=50\%$, still a hindered settling velocity is determined, while in reality this hindered settling velocity will be almost zero. Normal dense sands will have a porosity of about 40%, so $C_{vb}=60\%$. A fixed bed may have a porosity of 40%, but a sliding bed will have a higher porosity in between 40% and 50%. The porosities mentioned here depend on the type of sand, but are mentioned to give a feeling of the order of magnitude. An equation for hindered settling, that may work better is:

$$\frac{v_{th}}{v_t} = e^{-\beta \cdot C_{vr}^{1.25}} \cdot (1 - C_{vr}^{2 \cdot \beta}) \quad (17)$$

For small concentrations, this equation gives the same result as the original Richardson & Zaki (1954) equation, but for concentrations approaching the bed concentration, this equation approaches a zero-settling velocity. This would describe the bed behavior much better. So, for small concentrations this equation describes hindered settling, while for large relative concentrations approaching 1, the behavior is more close to consolidation behavior. The power β in this equation is equal to the original power β .

To determine the vertical location dependent hindered settling, the local concentration has to be known. In the zero step (index 0) the concentration profile is determined without local hindered settling, based on equation (17). The location dependent concentration is already part of the solution and should not be taken into account in the first iteration (correction) step. Now at each level in the pipe the corrected concentration gradient can be determined according to:

$$\left(\frac{dC_{vs,1}(r)}{dr} \right) = \left(\frac{dC_{vs,0}(r)}{dr} \right) \cdot \left(\frac{e^{-\beta \cdot C_{vr,0}^{1.25}} \cdot (1 - C_{vr,0}^{2 \cdot \beta})}{e^{-\beta \cdot C_{vr}^{1.25}} \cdot (1 - C_{vr}^{2 \cdot \beta})} \right) \quad (18)$$

The left-hand side is the corrected concentration gradient, the first term on the right-hand side the concentration gradient determined with equation (12). The second term on the right-hand side gives the correction factor according to the equation (17), but with a relative concentration dependent power. The power β is the Richardson & Zaki (1954) power with a value of 4.7 for very small particles and 2.4 for large particles. After

determining the corrected concentration gradient at each level in the pipe, the new concentration profile is found by integrating the concentration gradient from bottom to top.

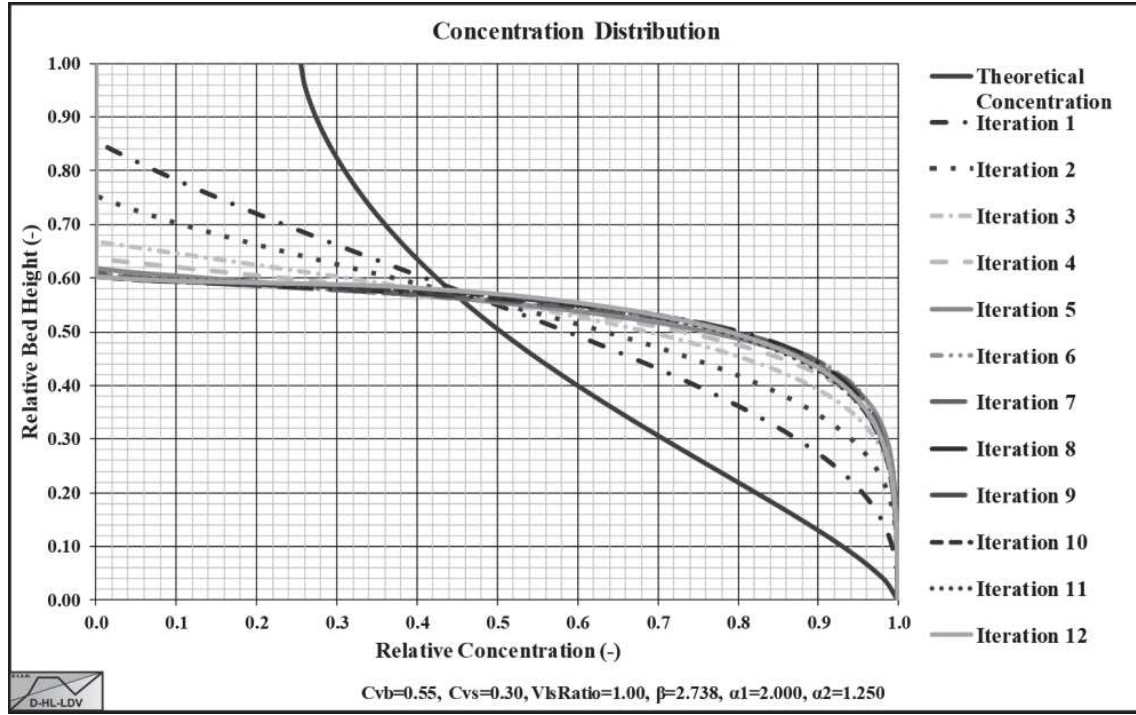


Fig. 1. Concentration profiles with and without local hindered setting (12 iterations).

It is assumed that the bottom concentration is unchanged. This process can be repeated a number of times until there is no significant change in the concentration profile. Here 12 iteration steps are used. For the next iteration steps (starting at index 2) also the location dependent relative concentration ratio is added, because it influences the concentration gradient, giving:

$$\left(\frac{dC_{vs,i}(r)}{dr} \right) = \left(\frac{dC_{vs,i-1}(r)}{dr} \right) \cdot \left(\frac{C_{vr,i-1}(r)}{C_{vr,i-2}(r)} \right) \cdot \left(\frac{e^{-\beta \cdot C_{vr,i-1}^{1.25}} \cdot (1 - C_{vr,i-1}^{2\beta})}{e^{-\beta \cdot C_{vr,i-2}^{1.25}} \cdot (1 - C_{vr,i-2}^{2\beta})} \right) \quad (19)$$

Using this new equation gives significant different concentration profiles. The concentration profile of a sliding bed with sheet flow is simulated well with this equation. The powers used in this equation (1.25 and 2β) are a first attempt and may be changed slightly in the future.

Fig. 1.1 shows a concentration profile for a line speed of 1 times the LDV and a relative concentration of about 50% of the bed concentration. The red line shows the concentration profile as determined with equation (17). This results in the 12 iteration steps, where above the bottom the concentration is increased, while at the top the concentration has decreased. It is clear from this figure that 12 iteration steps give enough convergence. At high

concentrations, the concentration profile at the top of the pipe may not be accurate, especially at small line speeds. This is caused by the high concentrations at the bottom of the pipe. Fig. 2 shows experimental data of Matousek & Krupicka (2014), matching the theoretical curves well.

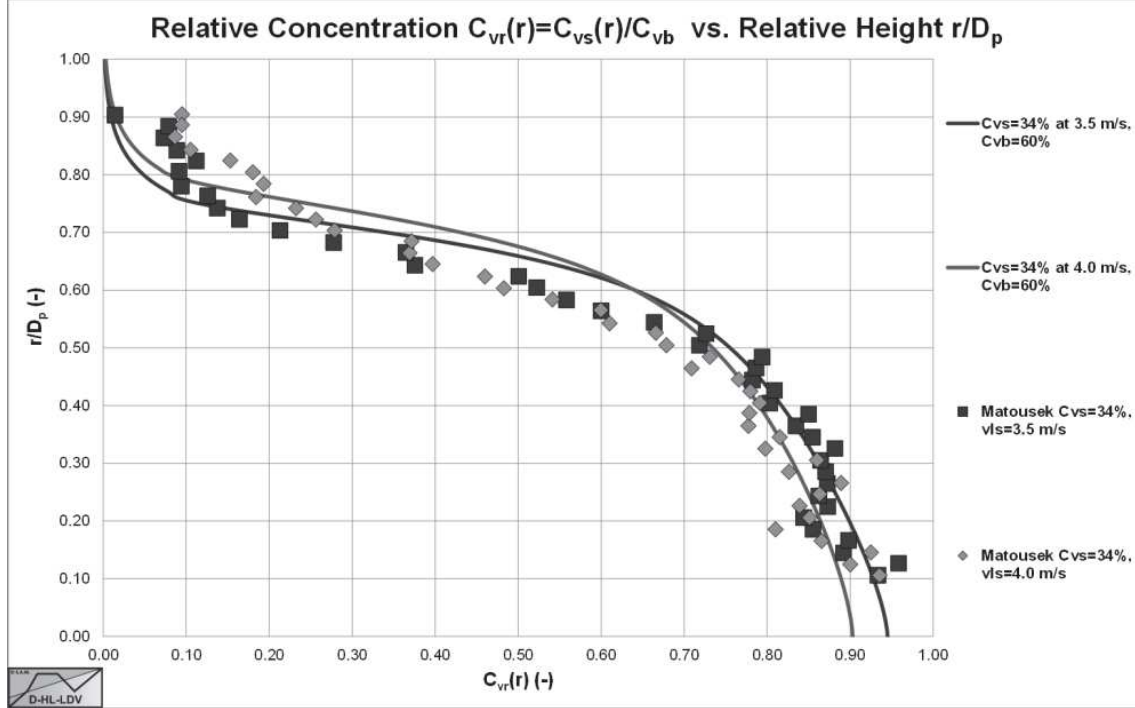


Fig. 2. Experiments of Matousek & Krupicka (2014) in a $D_p=0.1$ m pipe with $d=0.53$ mm particles, $C_{vs}=0.34$.

4. CONCLUSIONS

The theoretical concentration distribution profiles based on the advection diffusion equation may implicitly contain cross sectional averaged hindered settling, but not vertical coordinate dependent hindered settling. To add local concentration dependent hindered settling, first the hindered settling equation must be adapted, to match or almost match the no settling criterion in a bed. With the modified equation, the initial concentration profile can be determined. In a second step the local concentration gradient can be corrected for the local concentration and integrated to achieve a new corrected concentration profile. This can be repeated until the concentration profile between two step does not significantly change anymore. Usually 4-12 steps are sufficient.

The resulting concentration profile matches very well with experimental data and clearly shows a bed with a shear layer (sheet flow) on top for line speeds up to the LDV. For line speeds, much higher than the LDV the initial concentration profile is much steeper, requiring less correction.

A difficulty using this method is the determination of the bed concentration. For natural sands a dense sand may have a concentration of 60%, while loose sand may have a

concentration of 50%. When a bed is formed, probably the initial concentration will be closer to 50%, however decreasing the line speed may result in a higher concentration. This is still subject of further research.

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