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GLOBAL STABILITY OF CONCENTRATION PROFILES IN UNIFORM SEDIMENT-LADEN CHANNEL FLOW

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A famous quote of Landau and Lifschitz reads: "Yet not every solution of the equations of motion, even if it is exact, can actually occur in nature. The flows that occur in nature must not only obey the equations of fluid dynamics, but must also be stable". In this paper we examine the global stability of concentration profiles of fine sediment in uniform channel flow. We integrate the fully transient equations with a conservative numerical scheme to obtain the asymptotic states for a wide range of initial conditions. It is found that the steady state solutions are stable and are therefore likely to develop under experimental conditions. The decay rate of the disturbances is in line with the hydraulic time scale defined by the ratio of the channel height and the friction length. In combination with the averaged flow velocity it is possible to assess the adaptation length of the concentration profile along the channel slope. This information is important for the interpretation of measured concentration profiles that are assumed to represent a steady-state solution for benchmark purposes. In addition the role of the added mass force on the stability is studied. The role of this force seems limited for the experimental conditions studied in this paper.

KEY WORDS: sediment, turbulence, channel flow, stability, Rouse profile, hindered-settling, adaptation length

1. INTRODUCTION

Uniform sediment laden channel flow is an elementary validation case for sediment modelling techniques in hydraulic transport ranging from purely empirical approaches to advanced computational fluid dynamics models such as Kaushal et al (2012) and Goeree et al (2016). For this reason it is very important to have a fundamental understanding of this problem. The first reference goes back to Schmidt (1925) who derived an advection-diffusion equation for sediment in uniform channel flow. Rouse (1937) solved this equation analytically with appropriate closure of the turbulent diffusivity. Halbronn (1949) and Hunt (1954) improved the original equations of Schmidt (1925) by considering volume conservation of both phases. This correction is very important for high sediment concentrations. Van Rijn (1984) and Winterwerp et al (1990) incorporated the hindered-settling velocity of particles to the Hunt (1954) and Schmidt (1925) formulation respectively. Greimann and Holly Jr (2001) showed that it is possible to retrieve the Rouse (1937) profile form the two phase formulation derived in Greimann et al (1999). Keetels et al (2017) compared the advection-diffusion approach of Schmidt (1925), Rouse (1937), Van Rijn (1984) and Winterwerp et al (1990) with the two-phase model of Greimann et al (1999) and several formulations of the drag force in turbulent flow. They demonstrated that the Halbronn (1949) and Hunt (1954) corrections are not consistent with momentum conservation. The hindered-settling correction of Winterwerp et al (1990) can indeed be retrieved if it is assumed that the friction factor of particles in uniform channel flow is the same as the friction factor under terminal settling conditions. Concentration profiles obtained with the correction of Winterwerp et al (1990) were also compared with profiles obtained with an appropriate estimate for particle friction factor in turbulent channel flow. The numerical differences between both approaches are smaller than five percent. Keetels et al (2017) also found that the steady state concentration profiles could be obtained by integration of transient equations for the vertical sediment velocity for a wide range of initial conditions. This demonstrates that these profiles are stable and are therefore likely to develop under experimental conditions following the famous quote of Landau and Lifschitz: "Yet not every solution of the equations of motion, even if it is exact, can actually occur in nature. The flows that occur in nature must not only obey the equations of fluid dynamics, but must also be stable". This paper focus in more detail on the stability of these solutions by providing a modal analysis, study of the decay rate of the perturbations and extend the analysis by assuming different boundary conditions and involving the added mass forces in the transient equations.

2. PROBLEM DEFINITION

Figure 1 provides a more precise definition sketch of sediment in uniform channel flow. It is assumed that horizontal velocity of the solid U_S and fluid U_f depend on the vertical location but are constant in time and do not vary with the horizontal location. The channel height is also constant. The vertical velocity of the fluid V_f and solid V_s and the concentration Φ_s vary in time and in the vertical location. It is assumed that the initial concentration is perturb and the final concentration distribution is unknown. The question is whether a steady concentration distribution can develop under these condition for all possible initial profiles.

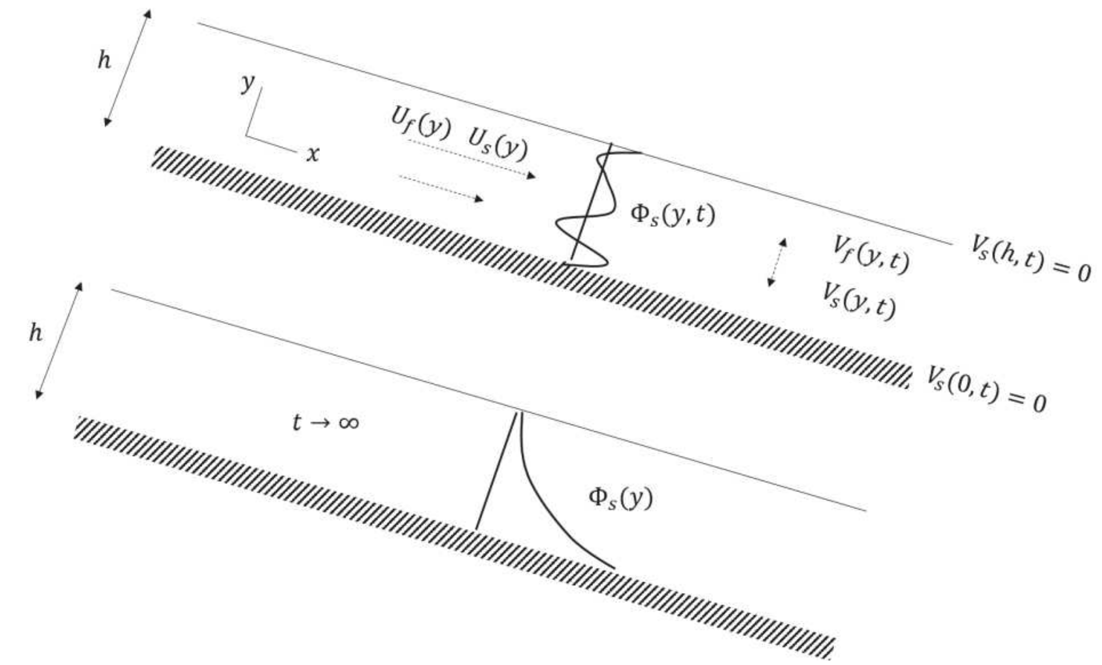

Fig.1 Definition sketch.

3. TWO-PHASE MODEL

Keetels et al (2017) casted the original two-phase formulation of Greimann et al (1999) in a convenient form to obtain transient solutions. The main novelty is that the transient pressure gradient is eliminated from the system of equations, while the solid concentration and velocity difference between both phases remain as unknowns. Here follows a brief description of the derivation of these equations and a few notes on the closures involved. From the mass balance it follows that:

$$\frac{\partial \Phi_s}{\partial t} + \frac{\partial \Phi_s \mathrm{V}_s}{\partial y} = 0 \tag{1}$$

By combining the momentum balances of each phase as derived by Greimann et al (1999) equipped with appropriate closures for the coupling forces with the mass balance Eq. (1) it is possible to derive an evolution equation for the velocity difference $V_l = V_f - V_s$

$$\left(1+\frac{\rho_f C_A}{(\Phi_s\rho_f+\Phi_f\rho_s)\Phi_f}\right)\frac{\partial V_l}{\partial t} = \frac{\rho_s-\rho_f}{\Phi_s\rho_f+\Phi_f\rho_s}|g| - \frac{1}{(\Phi_s\rho_f+\Phi_f\rho_s)\Phi_f\Phi_s}\left(\frac{\Phi_s\rho_s}{\tau_p}(V_l+V_d)+\rho_f C_A V_l \Phi_f\frac{\partial \Phi_s}{\partial t} + \rho_f C_A \frac{\partial \Phi_s \mathrm{V}_d}{\partial t}\right) \quad (2)$$

where ρ_s and ρ_f represents the density of the solid and fluid phase and C_A is the added mass coefficient, τ_p is the relaxation time of a solid particle and V_d is the drift velocity. The drift velocity represents the correlation between local regions of high/low vertical fluid velocity fluctuations and the presence/absence of solid particles (Greimann et al, 1999). This term can be modelled as a diffusive flux in homogenous turbulence, yielding

$$V_d = -\varepsilon_{yy}\frac{1}{\Phi_s\Phi_f}\frac{\partial \Phi_s}{\partial y} = -\varepsilon_{yy}\left(\frac{1}{\Phi_s}+\frac{1}{\Phi_f}\right)\frac{\partial \Phi_s}{\partial y} \quad (3)$$

Eq. (1) and Eq. (2) can be solved in conjunction with appropriate closures for the diffusivity and approximation of the particle relaxation time. The turbulent diffusivity is estimated as

$$\varepsilon_{yy} = u_* \kappa y(1-y/h) \quad (4)$$

where κ represents the Von Kármán constant and u_* is the friction velocity. The particle relaxation time is defined as,

$$\tau_p = \frac{\rho_s d_p^2 \Phi_f^{\beta}}{18 \upsilon_f \rho_f C_f} \quad (5)$$

where d_p is the particle diameter, β is a parameter to correct the particle friction factor C_f for concentration. If it is assumed that the friction factor of a particle in channel flow is the same as the friction factor during terminal settling conditions it holds that

$$\tau_p = \frac{\mathrm{V}_p^{\infty}\rho_s \Phi_f^{n-2}}{g(\rho_s-\rho_f)} \quad (6)$$

where V_p^{∞} is the terminal settling velocity of a single particle in stagnant water and n denotes the hindered-settling exponent of Richardson and Zaki (1954). It should be noted that Eq. (2) is only valid for τ_p that is smaller than the diffusive time-scale $\varepsilon_{yy}/\overline{v_s'^2}$. This corresponds with experiments with relatively fine sediment at some distance from the wall.

In Keetels et al (2017) it is shown that if Eq. (6) is valid, the steady state solution of system Eq. (1) and Eq. (2) can be expressed as

$$V_p^{\infty}\Phi_f^n\Phi_s + \varepsilon_{yy}\frac{d\Phi_s}{dy} = 0, \tag{7}$$

which implies that under that assumption the sediment balance consist of a hindered-settling flux and a diffusive flux in the same spirit as the original Schmidt (1925) / Rouse (1937) model for the dilute case.

4. NUMERICAL METHOD

Eq. (1) and Eq. (2) are intrinsically coupled and are strongly non-linear. An approximate analytical solution could be obtained by linearization of the solution around the steady state solution. In this way only small perturbations can be studied. Moreover, the boundary conditions at the wall and top of the channel are not periodic. Therefore, the perturb solution cannot be decomposed by standard Fourier series expansion. Other base function can be used, like the Chebyshev expansion but this makes interpretation of the results more complicated. A conservative numerical integration of Eq. (1) and (2) is more convenient to study the evolution of the concentration and vertical velocity profiles for all type of boundary conditions and large initial perturbations. The vertical direction is divided in a finite number of control cells. The cell centres contain the cell-averaged concentrations and the cell faces contain the vertical solid velocity. Integration of Eq. (1) yields an expression for the cell averaged concentration as a function of the solid flux at the faces of each cell. Eq . (2) gives the solid flux at the cell faces, with central interpolation for the concentrations and central differencing for the concentration gradients. For the faces at the upper and lower wall the no-flux boundary condition is imposed in a natural way. This yields a consistent and mass conserving integration scheme that is first order accurate in time and second order accurate in vertical grid spacing. As it is a one dimensional problem computational time restrictions are absent allowing full grid and time step convergence.

5. TRANSIENT CONCENTRATION PROFILES

Figure 2 shows both the steady state solution of Eq. (7) and transients solutions of the system of equations Eqs. (1) and (2) for an uniform initial profile. It concerns the experimental conditions S16 of Einstein and Chien (1955), with $h = 11.9$ (cm), $\bar{U} = 2.00$ (m/s) $U_* = 12.5$ (cm/s), $= 0.182$, $d_p = 274$ (μm), $V_p^{\infty} = 3.8$ (cm/s), $\rho_s = 2650$ (kg/m^3). It is observed that the transient solution converges gradually to the steady state solution of Eq (1) and Eq (2). This is an important observation as in many experiments an approximately uniform profile (fully mixed) is employed at the inlet of the channel and it is assumed that the profile gradually develops along the channel slope. Figure 2 supports this assumption provided that the horizontal fluid and solid velocities are unaffected by the vertical sediment distribution.

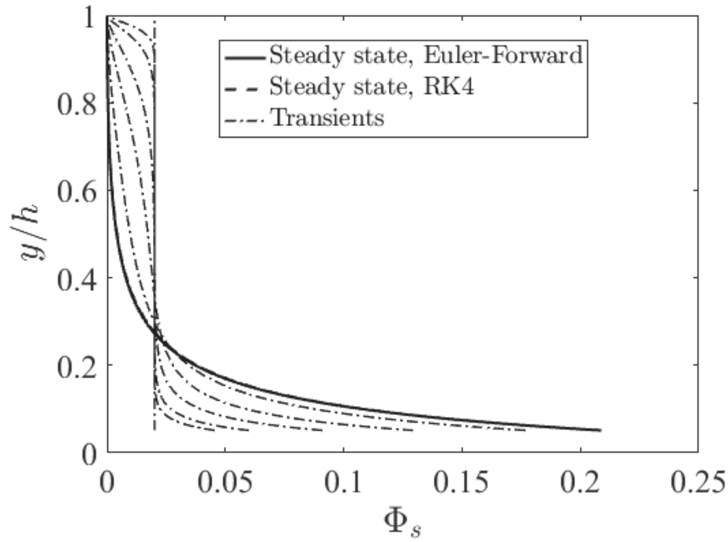

Fig.2 Steady state concentration profile obtained with two standard integration methods and transient solutions of Eq. (1) and Eq. (2) given at $tu_*/h =$ 0, 0.1, 0.2, 0.5, 1, 2, 10.

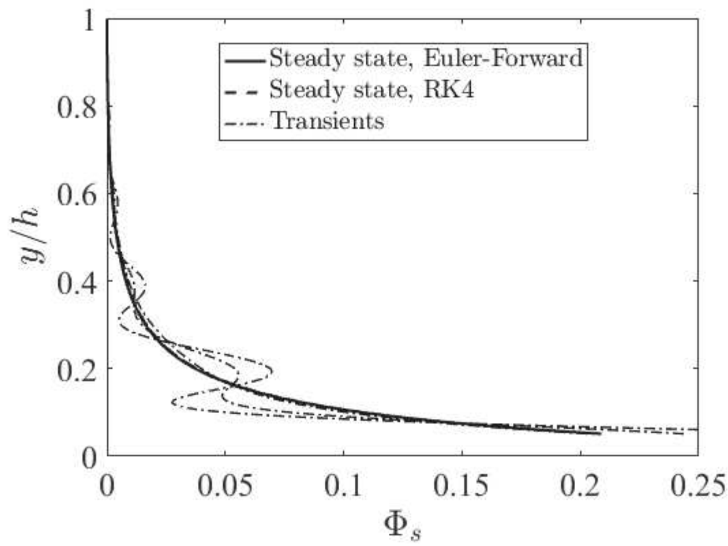

Fig.3 Steady state concentration profile obtained with two standard integration methods and transient solutions of Eq. (1) and Eq. (2) given at $tu_*/h =$ 0, 0.025, 0.1,5.

Figure 3 shows the transient solution for a perturbation around the steady state profile. The perturbation is generated as a modulated sinusoidal shape with wave number k=5 and is chosen such that the averaged concentration is the same as the steady state solution. Again it is observed that the transient solutions converge to steady state. This demonstrates that the steady state concentrations profiles of system of Eqs. (1) and (2) are stable and can be expected under experimental conditions.

6. MODAL DECAY ANALYSIS

Now it is interesting to study the decay rate for several modal initial disturbance. Figure 4 shows the decay rate of L_2 or energy norm of different modes with different wavenumbers k. The time axis is scaled with the hydraulic time unit h/u_*. It shows that the decay rate increases with the wave number. The energy in the lowest order disturbance is strongly reduced within one hydraulic time unit. The added mass terms in Eq. (2) have a small contribution to the decay rate of each mode.

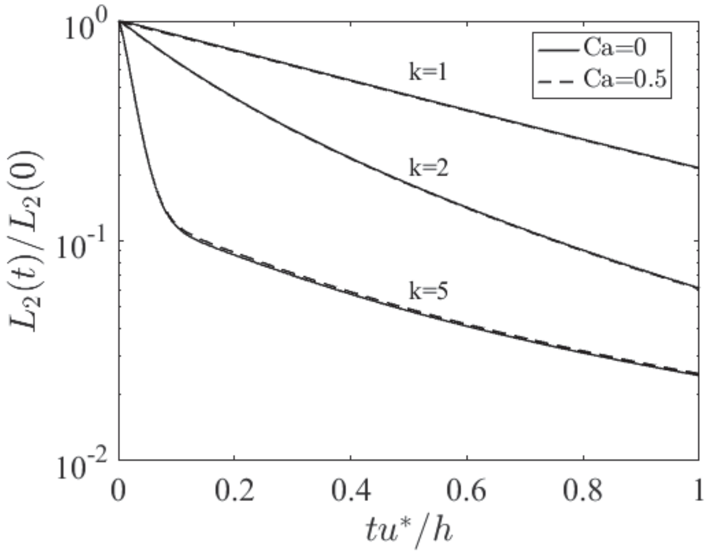

Fig 4 Decay rate of variance for different wave number of the initial disturbance.

It is also interesting to evaluate the decay rate in terms of horizontal displacement of the fluid along the channel slope. One hydraulic time unit times the average velocity ($\bar{U}h/u_*$) represents a length scale. From Figure 4 it can be deduced that disturbances at all length scales will be dissipated within a few hydraulic time or length units. For experiment S16 of Einstein and Chien (1955) it holds that the channel length $\sim 6\bar{U}h/u_*$, which suggests that their measured concentration profiles can be regarded as steady state/ fully developed. This observation is also consistent with the transient profiles shown in Figure 3.

7. CONCLUSION

From this analysis of transient profiles of concentration and vertical solid velocity it has been established that a steady state profile will develop independent of the initial sediment distribution. High wave number disturbances decay faster than low wave number disturbances. The decay rate of the lowest wave number is still in line with the hydraulic time unit. Comparison of the channel length and the adaptation length of the profile based on the friction velocity, average velocity and channel height, suggests that the fine sediment profiles provided by Einstein and Chien (1955) can be regarded as equilibrium profiles, which makes them very useful for benchmark purposes. This analysis is, however, restricted to the vertical momentum balances of both phases and assumes that the horizontal velocities and turbulence properties are frozen. In reality the directions are coupled and more complicated interactions due to vertical transport of horizontal momentum and density stratification might exist. For coarser sediment types collisional stresses and inertial transport terms in the momentum balance become important as well. These topics will be subject for further investigation.

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