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A SOLID-BODY MODEL FOR SWIRLING FLOWS

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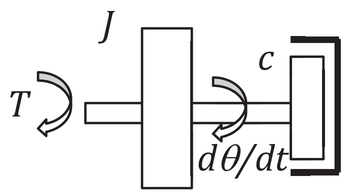
Away from duct walls, a particle-bearing liquid acts as a solid body in both axial and circumferential motion. A solid-body analogy can therefore provide an interesting, if controversial, model for swirling flows. The idea bridges fluid mechanics, solid-body mechanics and dynamical systems theory. Representing a liquid cylinder as a fluid flywheel with viscous friction yields a first-order system in downstream length. The model is compared to existing empirical/theoretical models of swirl decay and illustrated by reference to swirl duct examples.

1. INTRODUCTION

Visco-plastic particle-bearing liquids (most industrial slurries), and simple fluids beyond the laminar-turbulent transition, appear to flow through cylindrical pipes like solid cylindrical bodies moving at a constant axial and circumferential velocity. Kreith and Sonju (1965) described the solid-body phenomenon in circumferential flows, as did later researchers. Kitoh (1991) demonstrated that for a fixed Reynolds number a first-order exponential decay formula is consistent with the tangential momentum equation written in cylindrical polar co-ordinates. This was an important endorsement because it implied that computational methods applied to the conservation laws (*e.g.* Computational Fluid Dynamics, *CFD*) are consistent with the Solid-Body Model. The response of such a system to increasing or decaying swirl generation depends on the group τu, where τ is the time constant of the system and u is the axial velocity.

2. THE SOLID BODY MODEL

Reflecting on the simplified system dynamics of the analogy of a solid liquid cylinder, length one pitch (for one 360° rotation), subjected to a torque T (positive for a profiled swirl pipe, approaching zero for swirl decay) rotating at temporal rate $d\theta/dt$ …



$$T = J\frac{d^2\theta}{dt^2} + c\frac{d\theta}{dt} \qquad (1)$$

where J is the polar second moment of mass of the cylinder and c represents a damping coefficient dependent upon the area of the shearing surfaces.
Rewriting [1] so that the dependent variable is axial distance along the cylinder (z) and putting G=twist gradient $d\theta/dz$

$$T = J\frac{d^2\theta}{dz^2}\left(\frac{dz}{dt}\right)^2 + c\frac{d\theta}{dz}\left(\frac{dz}{dt}\right) = J\frac{dG}{dz}u^2 + cGu \tag{2}$$

where u = axial velocity. Dividing throughout by cu

$$\frac{T}{cu} = \frac{Ju}{c}\frac{dG}{dz} + G = \tau u\frac{dG}{dz} + G \text{ , where time constant } \tau = \frac{J}{c} \tag{3}$$

At this point we must depart from the concept of a solid shaft with a fixed torque, T, transmitted through its length. The fluid cylinder is quite distinct and torque varies lengthwise. We must transcribe equation [3] in terms of functions continuous in z.

$$G_D(z) = \tau u\frac{dG}{dz} + G \tag{4}$$

$G_D(z)$ drives the swirl by applying torque to the cylindrical fluid at its periphery.
The *complementary function*, $G_{cf}(z)$ or transient, is the solution to $\tau u\frac{dG}{dz} + G = 0$, *i.e.*

$$G_{cf}(z) = Be^{-\frac{z}{\tau u}} \tag{5}$$

The solution to equation [4], $G(z)$, has another (steady state) part, the *particular integral*, $G_{pi}(z)$, which depends on the driving function $G_D(z)$.

$$G(z) = G_{cf}(z) + G_{pi}(z) \tag{6}$$

Two cases of $G_D(z)$ are to be considered. The first case is the decay of swirling flow in a cylindrical pipe. The second case is the generation of swirl in a profiled duct.

2.1 $G_D(z) \to 0$:THE DECAY OF SWIRLING PIPEFLOWS

The driving function $G_D(z)$ in the case of the decay of a swirling pipeflow is a negative step from the initial swirl angle to zero. When $z=0$, $G(z)=G_o$, the initial swirl gradient. Equation [5] gives

$$B = G_o \;\; i.e. \;\; G(z) = G_o e^{-\frac{z}{\tau u}} \tag{7}$$

Halsey (1987) studied the swirl in clean water following a double elbow. His work was aimed at measurement devices for which swirling flow is disruptive. ISO 5167 specifies a 2° swirl-angle limit for measurement purposes and Halsey came up with an empirical law for its decay as follows.

$$\theta = \theta_o e^{-\frac{1.5fz}{D}} \tag{8}$$

where θ_o is the swirl angle at commencement, θ is the swirl angle at a downstream distance z, f is the friction factor and D is the diameter of the bore. Steenbergen and Voskamp (1998) arrived at an almost identical equation in terms of *swirl intensity, Ω,* instead of swirl

angle. Swirl intensity is defined as $\Omega = \frac{\int_0^R uwr^2 dr}{R\int_0^R u^2 r dr}$, where w = tangential velocity, r = radial dimension and R = internal bore radius. Ganeshalingham's work (1998) achieved close agreement with these models.
Differentiating [8] gives

$$\frac{d\theta}{dz} = G(z) = \theta_o\left(\frac{-1.5f}{D}\right)e^{-\frac{1.5fz}{D}} \qquad (9)$$

When $z = 0, G_o = \theta_o\left(\frac{-1.5f}{D}\right)$, so for the Halsey model

$$G(z) = G_o e^{-\frac{1.5fz}{D}} \qquad (10)$$

Equating exponents in equations [7] and [10], we have a first estimate of the time constant τ for decay of swirl and the length of the swirling wake τu

$$-\frac{z}{\tau u} = -\frac{1.5fz}{D} \quad \text{from which } \tau = \frac{D}{1.5fu} \text{ and } \tau u = \frac{D}{1.5f} \qquad (11)$$

For some purposes the point of 95% reduction in swirl angle (L_{95}) after a distance of $3\square u$ should be a useful approximation to total extinction. If swirl is a desirable property (to keep solids in suspension for example), *half-life* distance ($L_{50} = 0.6931 \times (\tau u)$) is a better concept. The level of swirl at Reynolds number of 100,000 in an industrial steel pipe can be assumed to have decayed to half its initial value after about 26 diameters using the Solid-Body model (see Table 1)

Table 1

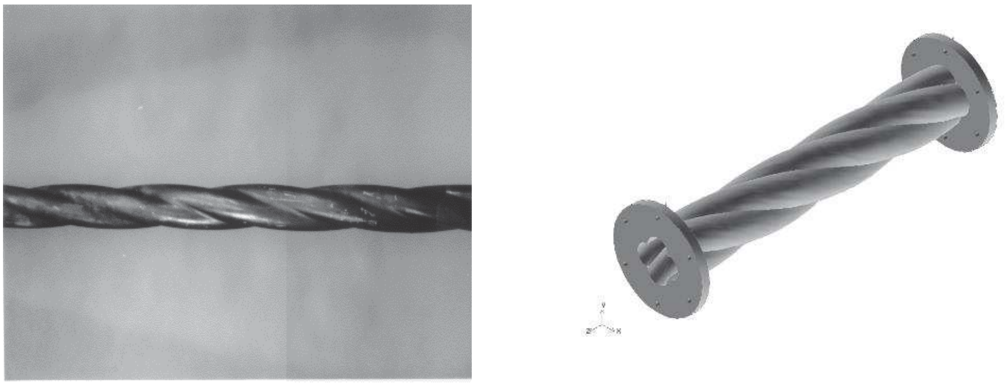
Solid Body Model:
Time / Distance Constants for a 50mm industrial steel pipe transporting clean water

Pipe velocity u	*Reynolds Number* Re	*Friction Factor** f	*Time Constant* τ	*Distance Constant* τu	*Extinction Length* L_{95}	*Half Life Length* L_{50}
m/s	-	-	s	m	m	m
1.5	75000	0.0205	1.0840	1.6260	4.8780	1.1271
2	100000	0.0185	0.9009	1.8018	5.4054	1.2489
2.5	125000	0.018	0.7407	1.8518	5.5556	1.2836
3	150000	0.018	0.6173	1.8519	5.5556	1.2836
3.5	175000	0.0175	0.5442	1.9047	5.7143	1.3203
4	200000	0.0175	0.4762	1.9048	5.7143	1.3203

* Estimated from the Moody Chart, Moody L.F. (1944), for which ...
Pipe diameter, D= *50mm*, pipe roughness height, $\varepsilon \sim$ *0.004 mm*, so $\varepsilon/D \sim$ *0.0008*

2.2 $G_D(z) = G_o$: GENERATION OF SWIRLING FLOW IN PROFILED DUCTS

There has been a broad body of research into swirl ducts for more than 100 years (Gordon, 1899). Lobed swirl pipes, the preferred design, were invented by Spanner (1940,1945) and are illustrated in Figure 1. The research has yielded data on the growth, and ultimate decay, of swirling flows downstream of these ducts from which the Solid Body Model can be tested.



(a) (b)

Figure 1: (a) Three-Lobed boiler tube invented by E.F.Spanner and used by Raylor (1998) in his experiments with particle-laden fluids. (b) Four-Lobed pipe used by Ganeshalingham (2002) and Jones and Ariyaratne (2007).

In equation [6] when $G_D(z) = G_o$ the complete solution (PI + CF) is given by

$$G(z) = G_{pi}(z) + G_{cf}(z) = G_o + Be^{-\frac{z}{\tau u}} \tag{12}$$

Applying the boundary condition $G(0) = 0$ gives $B = -G_o$

$$\text{So } G(z) = \frac{d\theta}{dz} = G_o\left(1 - e^{-\frac{z}{\tau u}}\right) \tag{13}$$

The 4-lobe pipe of Ganeshalingham (2002) had pitch/diameter ratio of 8. The pipe is illustrated in Figure 1. Note the full 360° rotation of the profile which implies that the length of the pipe is one full pitch or 0.4m. For a 50mm diameter pipe the spatial frequency would be given by

$$G_o = \frac{2\pi}{8\times 0.05} = 15.71\ radians/m \tag{14}$$

At a series of points at a fixed radial position within the core flow (*e.g. r/R = 0.7*), the demanded tangential velocity, ($\hat{w} = G_o \times (0.7 \times 0.025) = 0.28\ m/s$) yields a response, *w*, as a function of downstream distance.
The response of clean water to the swirl profile is shown in Figure 2. Fitting a system response to a step input is always a compromise between the early points (*z* =0 to 0.2m) and the long-term output. Overall, a best fit for the constant τu (63.2% of distance to reach final velocity) ≈ 0.1m, but if one weights the early points, a value of τu ≈ 0.09m is probably a more realistic fit. Either value is considerably smaller than the value obtained for the decaying flow in §2.1.
A partial explanation for the disparity in values of the group τu in the two cases is the shape of the velocity profile. Angular velocity in the swirl duct develops like a forced vortex with swirl developing from the *outside to the centre* in a "wall-jet" pattern which develops into a solid-body pattern over the length of the duct (see Figure 3).

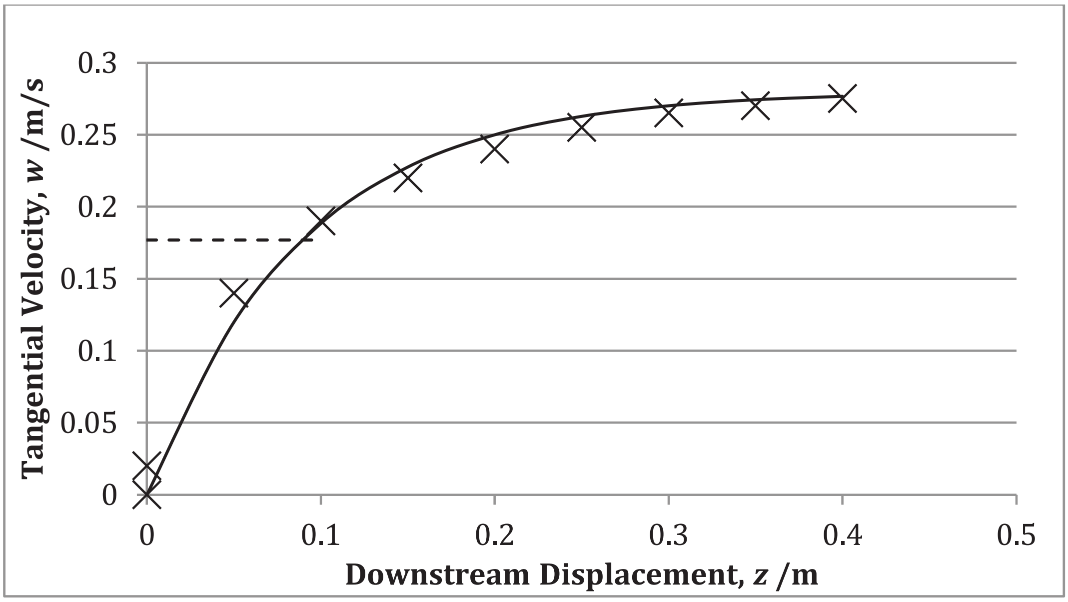


Figure 2: Response of tangential velocity of flow at a fixed radial position in a swirl duct. Axial velocity = 2 m/s. Distance constant $\tau u \approx 0.1$m, so the tangential velocity approaches its maximum 0.28 m/s after 0.3m.

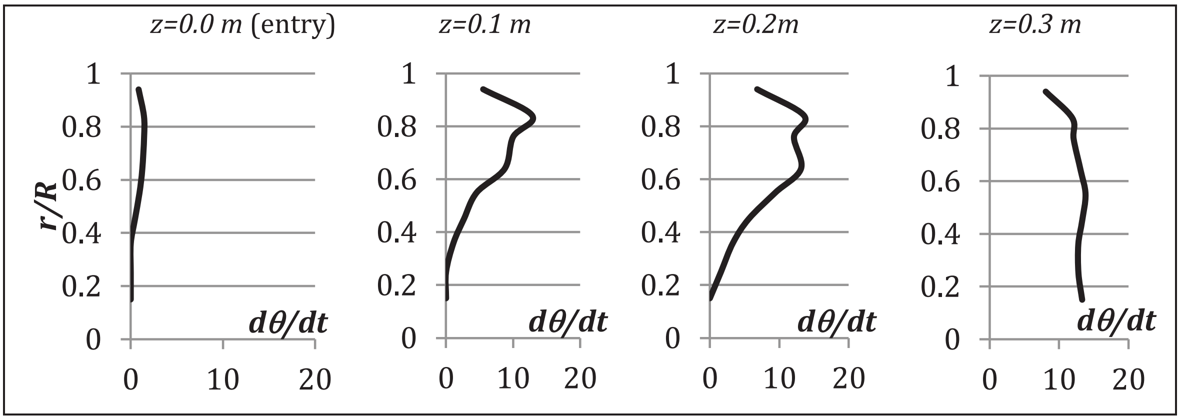


Figure 3: Angular velocity distributions within a 0.4m swirl duct.

3. DISCUSSION

Considering the wall jet in a swirl duct, the consequent reduction in the polar moment of inertia of the rotating mass, J, from the value for a solid cylinder has an important effect on the time constant (Time constant $\tau = \frac{J}{c}$). Similarly the area of the shearing surfaces, A, will be much greater, causing the coefficient of damping, c, to be considerably increased. The effect can be illustrated by a simplified geometric construction of the developing swirling mass in a pipe of length L ($\approx 4\times \tau u$).

$$J = J_{cyl} - J_{cone}$$

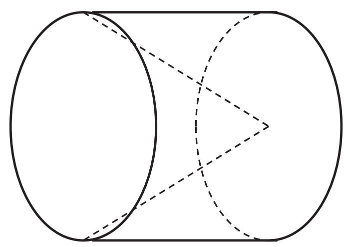
$$\therefore J = \frac{1}{2}\rho\pi R^4 L - \frac{1}{10}\rho\pi R^4 L = \frac{4}{5}J_{cyl} \quad (15)$$

$$A = A_{cyl} + A_{cone} = 2\pi RL + \pi R\sqrt{R^2 + L^2}$$

Putting *R=0.025m* and *L=0.4m*, $A = 1.46A_{cyl}$ (16)

Assuming $c \propto A$

$$\tau = \frac{J}{c} \approx 0.55 \times \frac{J_{cyl}}{c_{cyl}} \quad (17)$$



This geometric construction does not completely explain the 16 to 19-fold increase in the value of τu. A plausible explanation of the long period of decay in cylindrical pipe lies with the transmission of wall friction to the decaying swirl through the boundary layer. Whereas the swirl duct applies angled cusps into the maximal flow-rate, the friction torque applied to decaying swirl in cylindrical pipe is transmitted from a region of much reduced velocity.

The exit plane of a swirl-inducing duct illustrates the limits to the solid-body analogy. The swirl duct applies rotational acceleration to the fluid in the duct while at the exit plane rotational deceleration is applied as the swirl decays. There is no mechanical analogy for a single shaft with rotational acceleration at one end and rotational deceleration at the other. The fluid can no longer be regarded as a single solid body but as two solid bodies with a transitional pattern between them. Following exit from the profiled swirl duct, the central region is rotating like a solid body, while the periphery is suddenly free of profiled cusps. Initially therefore the flow develops like a free vortex, *i.e.* from the *central region to the outside*. Figure 4 shows the wall-jet pattern re-emerging briefly with its consequent reduction in time constant τ.

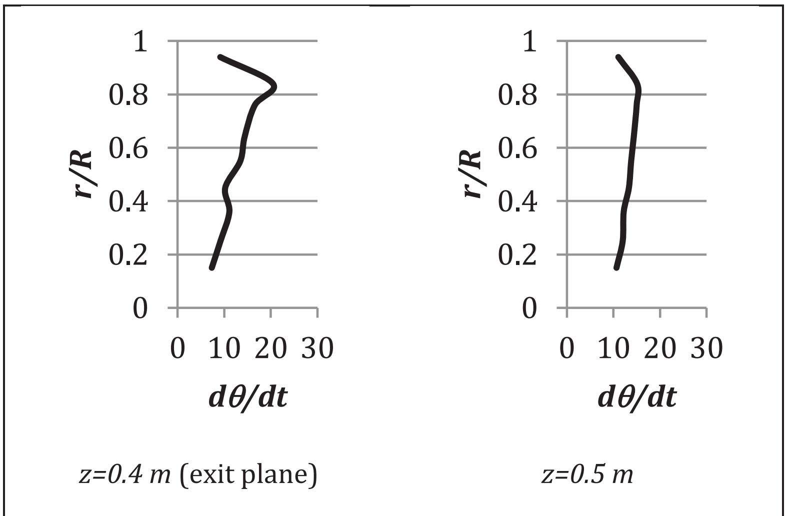


Figure 4: Angular velocity distributions at the exit plane and downstream of a 0.4m swirl-inducing duct.

Within 0.1m downstream the pattern has reverted to a solid cylindrical body. The effect on the decay of swirl is clearly shown by the work of Ariyaratne (2005). She shows that an optimized transition (which reduces the free vortex effect) brings the response close to the prediction of Steenbergen and Voskamp (1998) (see Figure 5).

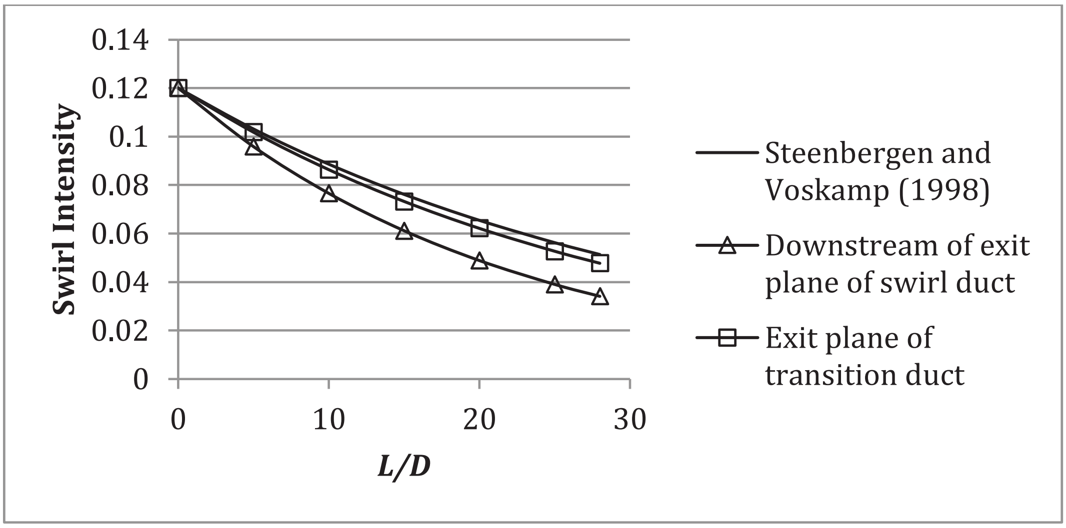


Figure 5: Effect of optimized transition piece after a swirl-inducing duct. The swirl intensity is calculated at the same position with or without the transition duct appended.
Data from Ariyaratne (2005) with thanks.

4. CONCLUSIONS AND RECOMMENDATIONS

The time constants and "distance constants" (τu) for flow in the two cases studied had contrasting values. In the case of swirl decaying in an industrial steel pipe, the distance constant (τu) was determined by comparing the predictions of the solid-body model to empirical measurements by Halsey (1987) and Steenbergen and Voskamp (1998). Their predictions led to the proposition that the distance constant was inversely proportional to the pipe friction factor only (see equation [11]). Consequently τu results for a 50mm pipe transporting clean water were a rather sluggish range from 1.6m to 1.9m (half-life 1.13m to 1.32m) over a typical range of axial velocities. However, in the case of swirl generated by a helically profiled duct, the distance constants (τu) were of the order of 0.1m, a factor of 16 to 19 times smaller. A second factor affecting the distance constant of the generation case was the "wall jet" angular velocity profile which would have a smaller time constant: possibly in the order of 55% of the value for the cylindrical solid (see equations [15], [16], [17]). A recommendation for future investigation is to research this apparent anomaly using new and historical experimental evidence, and wall roughness ranging from the very small to the mean boundary layer thickness and larger. Swirling flows are a little more difficult to predict than by using a simple exponential decay formula, but the solid-body model is a simple and useful tool to understand such flows.

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