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DISPERSION IN RESIDENCE TIME OF PARTICLES WITH POLY-DISPERSED SIZE FALLING THROUGH A LIQUID

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The residence time of a single, non-rotating spherical particle falling through a fixed distance in a stationary liquid is examined. The mass of the particle is taken to be a random quantity as a result of diameter being distributed according to the Log-Normal probability density function (particle density is assumed constant). In addition to weight, buoyancy and drag, the particle is assumed to be subject to random effects as it falls. The effect of both the dispersion in particle diameter and random disturbances in particle velocity on particle residence time are quantified. It is shown that the joint distribution in particle diameter and residence time can be approximated by a bivariate Log-Normal distribution. Such an approach permits the relative influence of systematic size dispersion and velocity fluctuations on residence time to be easily determined. It also permits the correlation between particle residence time and diameter to be expressed analytically. While the method is aimed at the motion of a single particle, it provides a fundamental basis to design systems based on falling particle-liquid interactions.

Keywords: Terminal velocity, Size dispersion, Velocity fluctuations, Residence time distribution.

1. INTRODUCTION

The motion of particles falling under gravity through a fluid is ubiquitous in chemical engineering operations. The time taken to fall through a certain distance, the residence time, is one of the most important parameters needed in the design of these operations. The determination of this quantity is not a trivial question both because particles are usually dispersed in size and for any given particle, the residence time is a non-deterministic quantity, Boschan et al. (2016). Particle size distribution it is usually found to be positively skewed and the Log-Normal distribution can be used to represent it, Jeantet et al. (2008). Residence time distribution is also invariably found to be right skewed. The aim of this paper is to provide an overall probabilistic framework to quantify the dispersion in residence time of particles arising both from a dispersion in particle size and simultaneously including the influence of random disturbances in particle velocity. While there are many possible causes of velocity fluctuations, their overall effect can be accounted for by a single dispersion parameter. Having a validated approach that incorporates both these contributory

factors to dispersion in residence time, means strategies to control residence time dispersion can be assessed or compared.

2. THEORY

2.1 DETERMINISTIC ANALYSIS

Assuming that a particle very rapidly achieves its terminal velocity, u_t in the fluid, the expression for residence time, τ for a particle falling through a distance, H is

$$\tau = \frac{H}{u_t} \tag{1}$$

For particles in the micron-sized range and above, the three dominant forces acting on a single, non-rotating, non-accelerating, spherical particle as it falls through a stationary fluid are gravity, buoyancy and drag force and terminal velocity can be obtained from their equilibrium.

$$u_t = \left(\frac{4\left(\rho_p - \rho_f\right) g\, d}{3\, C_D\, \rho_f} \right)^{0.5} \tag{2}$$

where ρ_f is fluid density, ρ_p particle density. Terminal velocity depends on particle diameter, d with the dependence being a function of how the drag coefficient depends on the Reynolds Number in the region of interest. The drag coefficient can be given a simple power law dependency on particle Reynolds Number where the accuracy of this approximation can be arbitrarily adjusted to any desired level of precision by refining the Reynolds Number range under consideration (to match the output of the accurate correlations).

$$C_D = \frac{a}{\mathrm{Re}^b} \tag{3}$$

where a and b are power law fitting parameters and their values depend on the Reynolds Number range of interest. Hence terminal velocity and accordingly residence time can be given a power law dependency on particle diameter.

$$\tau = \frac{H}{c_1 d^{c_2}} \qquad c_1 = \left(\frac{4\left(\rho_p - \rho_f\right) g}{3\, a\, \rho_f^{1-b}\, \mu_f^{b}} \right)^{\frac{1}{2-b}} \qquad c_2 = \frac{1+b}{2-b} \tag{4}$$

where μ_f is fluid dynamic viscosity.

2.2 PROBABILISTIC ANALYSIS

The relationship of equation 4 between residence time and diameter can be used to quantify the systematic dispersion in residence time resulting from a dispersion in particle size. Where particle diameter is represented by the Log-Normal distribution, the probability density function for diameter is

$$p(d) = \frac{1}{\sqrt{2\pi s_d^2}} \frac{1}{d} \exp\left[-\frac{1}{2} \left(\frac{\ln d - m_d}{s_d} \right)^2 \right] \tag{5}$$

where m_d and s_d are the parameters of the distribution corresponding to the mean and standard deviation of the transformed log-normal variates of diameter. The parameters m_d

and s_d can be obtained from the known (measured) values of the mean, μ_d and variance, σ_d^2 of particle diameter using:

$$m_d = \ln\left[\frac{\mu_d^2}{\sqrt{\mu_d^2+\sigma_d^2}}\right] \qquad s_d = \sqrt{\ln\left[1+\frac{\sigma_d^2}{\mu_d^2}\right]} \tag{6}$$

Given particle diameter is represented by the Log-Normal distribution, it can be shown that particle residence time will also be Log-Normally distributed according to

$$p(\tau) = \frac{1}{\sqrt{2\pi c_2^2 s_d^2}}\frac{1}{\tau}\exp\left[-\frac{1}{2}\left(\frac{\ln\tau - \left[-c_2 m_d - \ln\left(H/c_1\right)\right]}{c_2 s_d}\right)^2\right] \tag{7}$$

Equation 7 predicts the distribution in residence time for a population of particles with poly-dispersed size. However there is another effect that must be considered. In reality, it is found that owing to the real complexity of particle-fluid interactions, for any given particle size there is a range of possible residence times, Fornari et al. (2016). Particle velocity can be decomposed into a time-invariant deterministic component (terminal velocity) and a randomly fluctuating component, Mucha et al. (2004). Assuming the random component is zero-mean, Gaussian with an auto-correlation structure having a characteristic time scale much shorter than other relevant timescales of the system, then particle displacement corresponds to that of a Wiener process with drift, Stark & Woods (1994). Then for a particle of known diameter falling under gravity through a viscous fluid, the dispersion in residence time can be characterized by the Wald distribution which describes the first passage time of such a process. The dispersion in residence time is characterised by two parameters; terminal velocity u_t and particle diffusion coefficient (in the vertical direction), D, Nicolai et al. (1995). It can be shown that this extra aspect of dispersion in residence time is equivalent to amending the basic expression for residence time as

$$\tau = \frac{H}{c_1 d^{c_2}} q \tag{8}$$

where q is also a Log-Normally distributed random variable with a mean value of unity, termed the kinetic dispersion factor. The probability density function for q is

$$p(q) = \frac{1}{\sqrt{4\pi/Pe}}\frac{1}{q}\exp\left[\frac{-Pe\left(\ln q + 1/Pe\right)^2}{4}\right] \tag{9}$$

The distribution is defined by a single parameter, the particle Peclet Number which is a function of terminal velocity and particle diffusion coefficient.

$$Pe = \frac{u_t H}{D} \tag{10}$$

In turn the diffusion coefficient is the product of variance in particle velocity times a characteristic de-correlation time for velocity, t_c.

$$D = \sigma_u^2 t_c \tag{11}$$

Hence equation 8 enables the complete distribution in residence time to be treated as the product of two separate effects: one resulting from the systematic dispersion in particle size and the second being due to random fluctuations in particle velocity. It can be shown that residence time will also continue to be Log-Normally distributed and its probability density function will be

$$p(\tau) = \frac{1}{\sqrt{2\pi\, s_\tau^2}} \frac{1}{\tau} \exp\left[-\frac{1}{2}\left(\frac{\ln\tau - m_\tau}{s_\tau} \right)^2 \right] \tag{12}$$

where the parameters of the distribution m_τ and s_τ are obtained from

$$m_\tau = \ln\left(\frac{H}{c_1}\right) - c_2 m_d - \frac{1}{Pe} \qquad s_\tau^2 = c_2^2 s_d^2 + \frac{2}{Pe} \tag{13}$$

Hence statistics such as mean residence time and variance in residence time will be given as

$$\mu_\tau = e^{\left[m_\tau + \frac{s_\tau^2}{2} \right]} \qquad \sigma_\tau^2 = \left(e^{s_\tau^2} - 1 \right) e^{2m_\tau + s_\tau^2} \tag{14}$$

Note the probability density function of equation 12 converges to that of equation 7 as the fluctuations in particle velocity become insignificant and residence time dispersion solely results from the particle size dispersion. Furthermore because particle residence time, τ and diameter, d can be represented by the bivariate log-normal distribution as

$$p(d,\tau) = \frac{1}{2\pi\, s_d\, s_\tau \sqrt{1-\zeta^2}} \frac{1}{d} \frac{1}{\tau}$$
$$\exp\left[-\frac{1}{2\left(1-\zeta^2\right)} \left\{ \left(\frac{\ln d - m_d}{s_d} \right)^2 + \left(\frac{\ln\tau - m_\tau}{s_\tau} \right)^2 - 2\zeta \left(\frac{\ln d - m_d}{s_d} \right) \left(\frac{\ln\tau - m_\tau}{s_\tau} \right) \right\} \right] \tag{15}$$

where the correlation parameter, ξ is solely a function of s_d and Pe. With equation 15, the degree of correlation between residence time and particle diameter can be obtained.

3. MATERIALS & METHODS

A series of experiments were conducted to estimate the magnitudes of input data for the theoretical expressions and to validate the predicted output. The focus was on low Reynolds Number systems. Six sets of spherical beads were assembled with a known size distribution. These included four sets of glass beads with nominal diameters 1.17 mm, 3.16 mm, 5.02 mm and 5.91 mm all with a measured density of 2603 kg/m^3. Also there was a set of 5.86 mm diameter Nylon beads (measured density1150 kg/m^3) and 5.86 mm ABS Plastic beads (measured density 1929 kg/m^3). The sphericity and particle size distribution of each set of beads was measured. The behavior of the beads in three liquids were investigated; Silicone Oil (measured density and viscosity 970 kg/m^3 and 0.227 Pa s respectively), Gear Oil (measured density and viscosity 880 kg/m^3 and 0.091 Pa s respectively) and Paraffin Oil (measured density and viscosity 876 kg/m^3 and 0.104 Pa s respectively). For each particle/liquid combination, 50 beads were dropped individually within a cylinder containing the fluid (diameter 100 mm and working height 700 mm). Residence time was

recorded. In addition, the motion was analysed with a high speed camera working at 200 frames per second to quantify the variance and auto-correlation structure of particle velocity.

4. RESULTS

4.1 DETERMINISTIC RESULTS

Table 1 displays the experimentally measured and theoretically predicted values for terminal velocity for each particle-fluid system that was examined.

Table 1

Terminal Velocity

Fluid	Particle Type	Diameter (mm)	Terminal Velocity (mm/s)		Reynolds No.
			Experimental	Theoretical	
Silicone Oil	Glass A	1.17	3.29	3.12	0.02
Silicone Oil	Glass B	3.16	22.0	20.64	0.30
Silicone Oil	Glass C	5.02	50.0	45.30	1.07
Silicone Oil	Glass D	5.91	69.3	58.94	1.75
Gear Oil	Glass B	3.16	63.7	51.51	1.95
Silicone Oil	Nylon	5.86	7.45	7.04	0.19
Gear Oil	Nylon	5.86	30.95	25.10	1.75
Silicone Oil	ABS	5.86	37.8	34.22	0.95
Paraffin Oil	Nylon	5.86	17.9	18.65	0.88

Theoretical terminal velocity was calculated using the power law approximation method and applying a correction for wall effects. Reynolds Number (based on experimental velocity) is also shown. The level of agreement is good with the theoretical value generally being within 10 % of the predicted value. Reynolds Numbers are in the region from 0.02 to 2.0. Note magnitudes of the drag coefficient fitting power law parameters a and b will be close to the theoretical Stokes values of 24 and 1 respectively for the low Reynolds Number prevailing in this study.

4.2 PROBABILISTIC RESULTS

Table 2a quantifies the size-dispersion input data summarizing the distribution in particle diameter for the different sets of beads together with the corresponding Log-Normal density function parameters m_d and s_d. It can be seen that for most sets of beads, the particle size distribution is tightly controlled with a low standard deviation to mean ratio.

Table 2a

Particle diameter dispersion

Particle	Diameter Statistics		Diameter Log-Normal Parameters	
	Mean (m)	Std. Dev. (m)	m_d	s_d
Glass A	1.172×10^{-3}	0.10×10^{-3}	-6.753	0.085
Glass B	3.161×10^{-3}	0.18×10^{-3}	-5.758	0.057
Glass C	5.02×10^{-3}	0.03×10^{-3}	-5.295	0.006
Glass D	5.91×10^{-3}	0.08×10^{-3}	-5.131	0.014
Nylon	5.86×10^{-3}	0.02×10^{-3}	-5.140	0.003
ABS	5.86×10^{-3}	0.016×10^{-3}	-5.139	0.003

Figure 1 illustrates a measured velocity versus time history for the Nylon bead falling in Silicone oil showing the significance of the fluctuations. It is these fluctuations combined with the dispersion in particle size that produces the variability in residence time. Table 2b lists the input data needed for evaluation of the parameters associated with the probabilistic analysis; specifically the measured diffusion coefficient, the Peclet Number and the parameters for the residence time distribution. A low value for the Peclet Number is associated with low terminal (mean) particle velocity and a large amount of velocity fluctuations. The diffusion coefficient is obtained by a time-series analysis of the random component of particle velocity to obtain σ_u^2 and t_c needed for equation 11. Finally table 3 contains the experimentally measured mean and standard deviation in residence time together with the corresponding theoretically predicted values.

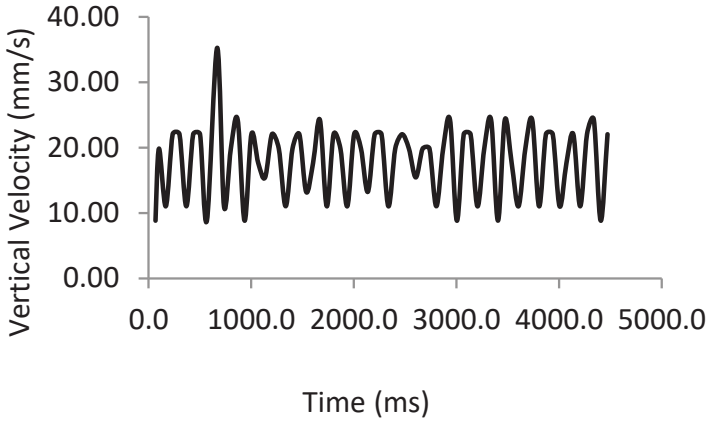

Fig 1 Experimental particle velocity versus time

Table 2b

Input dispersion parameters to calculate residence time

		Diffusivity	Peclet	Res. Time Log-Normal Parameters	
Fluid	Particle	D (mm^2/s)	Pe	m_τ	s_τ
Silicone Oil	Glass A	3.66	629.13	5.3897	0.180
Silicone Oil	Glass B	65.23	236.23	3.4492	0.137
Silicone Oil	Glass C	227.20	154.05	2.6169	0.120
Silicone Oil	Glass D	348.33	139.28	2.3336	0.132
Gear Oil	Glass B	162.81	273.61	2.5342	0.129
Silicone Oil	Nylon	41.23	126.55	4.4602	0.130
Gear Oil	Nylon	147.09	147.27	3.1882	0.129
Silicone Oil	ABS	200.60	132.04	2.8783	0.130
Paraffin Oil	Nylon	109.28	114.68	3.4854	0.130

Table 3

Experimental and theoretical residence time dispersion

System	Experimental Res. Time		Theoretical Res. Time	
	Mean (s)	Std. Dev. (s)	Mean (s)	Std. Dev. (s)
Silicone Oil – Glass A	212.7	21.7	222.71	40.39
Silicone Oil – Glass B	31.8	2.84	31.77	4.36
Silicone Oil – Glass C	14	0.158	13.79	1.66
Silicone Oil – Glass D	10.1	0.122	10.41	1.38
Gear Oil- Glass B	11	0.634	12.71	1.65
Silicone Oil-Nylon	93.92	5.88	87.23	11.35
Gear Oil – Nylon	22.62	0.9	24.45	3.18
Silicone Oil - ABS	18.5	0.298	17.93	2.33
Paraffin Oil – Nylon	39.1	1.79	32.91	4.28

The overall level of agreement is good. The predicted level of mean residence time is close to the experimentally measured value because predicted and measured terminal velocity are close in magnitude. The magnitude of dispersion in residence time is captured less accurately with the model consistently over-predicting the standard deviation. However the pattern of dispersion between the different fluid-particle systems is reflected in the results. The reason for the discrepancy lies in some shortcomings of the terminal velocity equation (the deterministic model) and certain simplifications in the probabilistic approach. Note the standard deviation for the first entry in table 3 is high reflecting the very long mean residence time. Characteristically for the above systems approximately two thirds of the variability in residence time is attributable to size dispersion whilst the remaining third results from fluctuations in velocity.

5. CONCLUSIONS

The total dispersion in residence time depends on the amount of dispersion in particle diameter (quantified by the size of s_d) and also will depend on the magnitude of the randomness in particle motion as quantified by the Peclet Number. As the system Peclet Number decreases reflecting higher levels of particle velocity fluctuations, random dispersion in residence time increases and as a corollary, a decreasing proportion of the variability in residence time is due to the variability in particle size and so a focus on size dispersion and attempts to minimize it become less rewarding in terms of process control. The approach here can be shown to correspond to the standard approach for calculation of residence time, Levenspiel (1999). Further work is needed to check if the decomposition of residence time adopted here will be valid outside the Stokes regime and to extend the methodology to a population of particles falling simultaneously in a vessel.

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